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THE ROOF STRUCTURES OF THE NEW SPORTS ARENA IN ATHENS

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SUMMARY

The principal purpose of the present article is to illustrate the various phases of the project, and to examine the roof structures of the new Sports Arena in Athens.

A general method of calculation for cable net structures with elastic border has been developed, considering the material and geometrical non-linearity.

Final verifications with the method of finite elements on the true structure of the border ring has been carried out.

1. - INTRODUCTION

In 1976 the Secretary General of Sports in Greece announced the competition for an urban design and architectural project regarding the new Sports Center at Phaleron to be built in an area of 250,000 square meters along the Athenian Coast near Pireo.

The succeeding year, the first price was given to the team of the Architecture and Urban design Studio of T. Papayannis and Associates. In the same year, 1977, the work was initiated with the preloading of the ground and the successive pile-drilling for the foundation.

The Sports Center at Phaleron was designed as requested in the announcement, to create an open green space and the possibility to locate different buildings principally:

- A Sports Arena for 15,000 spectators.
- An Olympic-sized swimming pool.
- A harbor for marin-related sports.
- A gymnasium for training.
- A Congress-Hall.

The object of the paper presented, is to illustrate some relevant problems manifested during the schematic and working design phase of the Sports Arena, principally concerning the roof structure.

In fig. 1 and fig. 2 one can observe the photos of the model of the global project.

In fig. 3 a-c the plan , the front-view and a section of the Sports Arena of Athens, object of the present paper, are illustrated.

2. - THE STRUCTURES OF THE SPORTS ARENA

The principal objective of the present paper is the illustration of the various phases of planning and statical analysis of the roof structure of the Sports Arena.

The roof structure consists of an orthogonal rope net with constant mesh in plan of 4 x 4 M, whose average surface can be defined geometrically as a saddle surface with total negative curvature, very similar to that of a hyperbolic paraboloid. The rope net is externally delineated by a border ring in prestressed reinforced concrete, which forms the anchorage structure of the cable net structure.

The structure of the anchorage ring is defined geometrically by the intersection between a circular vertical cylinder of diameter 113.96 M, which, on the other hand, is the maximum free span of the cable net structure, and a hyperbolic paraboloid co-axial to it.

S1

The maximum gradient between the points of the border line, defining the anchorage structure, is 12.3 M equal to about 11% of the free span.

The cable structural distribution in plan has been definitely decided according to a net mesh of 4 x 4 M after a careful analysis of the costs, in which, both, the cost of supply of the roof materials, and the relative cost of the operations relating to the erection and the setting in tension of the cable structure, have been considered.

In Table 1, the result of the comparative economic analysis between the two solutions of net mesh 4 x 4 M and 2 x 2 M in plan , is summed up. Considering as comparative unit-price cost the solution with net 4 x 4 M, from Table 1 it is possible to deduce that, as far as the total cost of the supply of the roof materials are concerned, the two solutions are equivalent. One, nevertheless has a tangible diversity examining the costs of erection and the operations, as regards the pre-stressing of the rope-net structure.

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S1

In fact, with a net mesh of 4 x 4 M, alghough the consumption of steel for the ropes and the cost of the covering remains about the same, the erection operation of the structure joint-locks is 4 times less; while the erection of the ropes and the stretching operation are halved.

sebbene

+piccolo

TABLE 1

Solution	Rope furniture	Roof covering (furniture, erection)	Erection and prestressing of the net
2 x 2	1.05	0.923	2 ÷ 2.5
4 x 4	1	1	1

S1

In the final solution the bearing and stabilizing ropes turn out to be 60 mm and 46 mm in diameter respectively.

portando

The spiral galvanized ropes adopted for the cable net structure are formed by 127 elementary wires with $\sigma_{ult} \geq 160$ kg/mm^2. A necessary sequence of prestressing has been given to the ropes in order to stabilize the elasticity modules which turns out to be $1.65 \cdot 10^6$ kg/cm^2.

The roof covering consists of corrugated galvanized sheet, pre-painted internally, 75 mm high and suitably insulated and water-proofed - it is fixed to the ropes by means of cadmium steel U-bolts.

The border structure, to which is anchored the rope net, consists of a perimetrical box-section ring in pre-stressed reinforced concrete that follows the movement of the border curve. This ring rests on 32 simple supports with a vertical bound.

S/

The supporting system is also equipped with hydraulic jacks, which allow the displacements due to elastic deformations and hinder the eventual movements of the rigid body of the ring.

S/

The supports are placed in relation to the frames supporting the stands. These frames, which are in plan of radial pattern, concentric as regards the centre of the construction, are also made of pre-stressed reinforced concrete and are variable height in order to follow altimetrically the movement of the border ring.

S/

The foundations of all these frames, connected to each other, rest on piles of 1.2 M diameter and about 30 M deep.

The track and all the accessory spaces are independent from the structures hereabove considered and rest directly on the dynamically consolidated ground.

3. - THE COVERING STRUCTURE

3.1. Analysis of loadings

3.1.1. Cable structure

		Load-Code
Own weight of the ropes	9 kg/M^2	[a]
Dead load of the covering and additional permanent loadings	36 kg/M^2	[b]
Snow loading	65 kg/M^2	[c]
Wind loading (c = -0.8)	110 kg/M^2	[d]

S/

3.1.2. Border ring

Dead weight of the ring	21.6 TON/M	[e]
Pre-stressing variable between two consecutive elements of the ring	(see Table 2)	[f]
Additional permanent loadings	1.4 TON/M	[g]
Accidental loadings	4.58 TON/M	[h]

S/

TABLE 2

Element number	1	2	3	4	5	6	7	8	9
Prestressing axial force (TON)	3350	3350	3000	2300	1250	2600	3500	3950	3950
Horizontal eccentricity (M)	0,93	0,93	0,775	0,465	0	-0.2	-0.3	-0.4	-0.4
Vertical eccentricity (M)	1.08	1.08	0,9	0.54	0	0.5	0.84	1.04	1.01

S/

3.2. Loading conditions

scelto

The combinations of the loadings, chosen leading to the definite statical verifications of the covering structures, have been determined according to the research of the maximum stresses and deformations, and also according to the various constructive phases, in accordance with a pre-set time schedule.

In accordance with the time-schedule illustrated in Table 3, the influence of the reologic deformations E(t) of the perimetrical ring has been duly considered for the various possible combinations of loading.

TABLE 3

Month	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Ring Performance	▥	▥	▥	▥					
Ring prestressing					▥	▥			
Cable-structure Erection					▥	▥	▥		
Cable-structure pre-tension							▥	▥	
Covering performance									▥

The static loading combinations considered are:

1) $\boxed{e}$
2) $\boxed{e} + \boxed{f}$
3) $\boxed{e} + \boxed{f} + \boxed{a}$
4) $\boxed{e} + \boxed{f} + \boxed{g} + \boxed{a} + \boxed{b}$
5) $\boxed{e} + \boxed{f} + \boxed{g} + \boxed{h} + \boxed{a} + \boxed{b} + \boxed{c}$
6) $\boxed{e} + \boxed{f} + \boxed{g} + \boxed{a} + \boxed{b} + \boxed{c}$

Associated to the conditions of maximum loading 5) and 6) the thermal variations have been considered in accordance with the following groups:

GROUP I

Difference in temperature between the inferior and superior faces of the ring

$$t_i - t_s = -15\ °C$$

Difference in temperature between the internal and external faces of the ring

$$t_i - t_e = -15\ °C$$

Difference of the temperature acting on the cable-structure:

$$\Delta T = -10\ ^\circ C$$

GROUP II

$$t_i - t_s = 15^\circ C$$

$$t_i - t_e = 15^\circ C$$

$$\Delta T = +10\ ^\circ C$$

4. - THE STRUCTURAL SCHEME AND THE CALCULATION METHOD

The position of the theoretical anchorage points of the cable structure is expressed analytically by:

$$\begin{cases} \dfrac{6.15}{56.98^2}\,(x^2 - y^2) + 28.74 = z \\ x^2 + y^2 = 56.98^2 \end{cases} \qquad (1)$$

Sl

The geometrical co-ordinates of the discrete anchorage points of the ropes are so determined on the ring, according to a Cartesian axes system, as indicated in fig. 4, considering that a constant net mesh of 4 x 4 M in plan has been adopted as a definite solution, as already described.

In the space, the structure of the anchorage ring is defined by moving the space frame, which defines the external contourn of the section, by resting on the generatrix determined by (1), keeping the plan containing this section vertical. The moments of inertia and section area are variable in consequence of the modification of the thicknesses of the walls of which it consists. In fig. 5 the 4 standard sections, which form the various elements between 2 consecutive supports of the ring and the relative moments of inertia and section area can be seen.

Sl

Sl

Taking advantage of the 2 symmetry axes, only a quarter of the ring, which has been schematized as a space frame, has been considered. The joints of this space frame have been defined principally, in correspondence to the rope anchorages, at the beginning and end of the variation of the ring section and in correspondence to its supports, which consist of the stand frames. These supports elastically and non-elastically yielding are eccentric with regard to the ring barycentric axis.

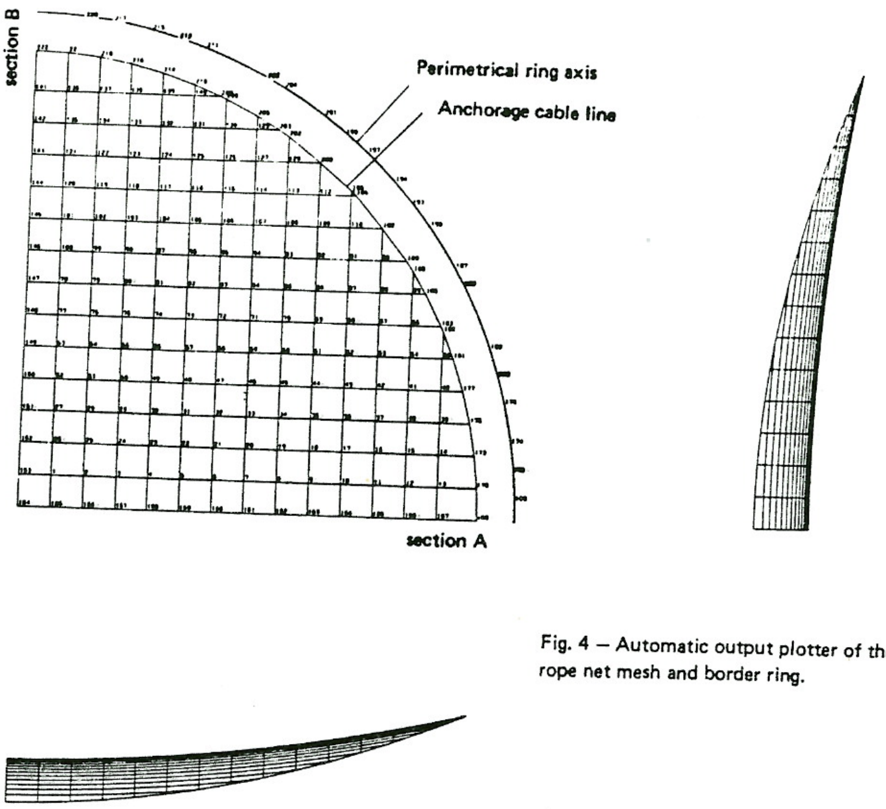


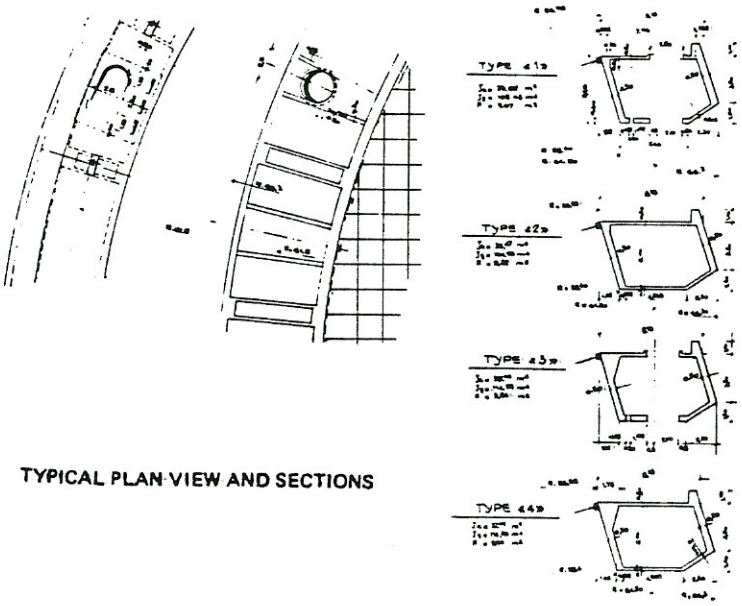
Fig. 4 – Automatic output plotter of the rope net mesh and border ring.

The rope anchorages are located at the ring upper plate (see fig. 5).

The cable structure is outlined as a system of bar-joints with the possibility of transmitting only tensile forces ($S > 0$) and consists of 167 internal joints having 3 degrees of freedom, 130 border joints and 385 bar elements.

The structural scheme of calculation foresees the elastic interaction of the cable structure with the border ring for all the loading conditions.

The organisation of the global programme of calculation foresees thus 3 principal groups (Routines) of resolution, which consists of:



TYPICAL PLAN VIEW AND SECTIONS

Fig. 5

- Programme RETE, for the research of state 0 .
- Programme TENSO, for the action of loadings on the cable-structure.
- A programme of space frame analysis, for the ring structure.

The programme RETE, given the interaction between the cable structure and the anchorage structure, forecasts the possibility of obtaining the state 0 . (The geometrical-tensional state of definition), taking into account the elastic deformation of the border ring.

In the research of the equilibrated geometry of the bar-joint set, the mathematical model of the rope net, is found by solving the non-linear equation system represented by the vectorial expression as:

$$\sum \vec{S}_{ki} + \vec{P}_k = 0 \quad , \tag{2}$$

written for all the internal points.

The elastic deformation state of the border structure, calculated by a space frame analysis programme is considered by an iterative procedure (see: Definition of state 0).

The action of the loadings on the cable-structure is elaborated by the programme TENSO, which solves the problem of the non-linear elastic equilibrium represented by:

$$[k]\{\delta\}^{r} = \{P\} - \{P^{*}\}^{r-1} \qquad (3)$$

where:

$[k]$ = stiffness matrix of the net structure in phase 0;

$\{\delta\}^{r}$ = vector of unknown displacements at the r^{th} iteration;

$\{P\}$ = vector of applied loadings at the joints;

$\{P^{*}\}^{r-1}$ = vector of non-linear therms at the r^{th-1} iteration.

The resolution method of the (3) is connected to the stiffness method and it is semi-incremental as regards the application of the loadings, the material non-linearity and the interaction with the border structure; on the contrary is iterative as regards the geometrical non-linearity (see loading states).

After having obtained, from the various loading combinations, the heaviest stress-states for the cable structure and for the border structure, some tensional verifications have been carried out, by means of the use of the finite elements technique (see verifications states).

5. - STATE " 0 "

The combination of loadings N.3 has been chosen for the definition of the state 0 of the structural system: border ring and cable structure.

Considering in the first moment the surface determined by the rope mesh like an hyperbolic paraboloid, the horizontal components of the stresses in the bearing and stabilizing ropes have been fixed in the pre-tension state in:

$$H_b = 58.19 \text{ TON}$$

$$H_s = 61.53 \text{ TON}$$

Once the values of the horizontal components at the rope anchorages have been determined, the state 0 of the ring-net system has been found considering the interaction of the rope net and the anchorage ring. Starting from a first calculation phase, considering the ring in a non-deformed position, an iterative process has been carried out between the research calculation of state 0 of the net and the deformation of the perimetrical ring.

The deformation of the ring, produced by the pre-tension forces in the ropes anchored to it, changes the geometrical conditions of the contour for the research of the equilibrated geometry of the bar-joint set.

The geometry of the net is now modified in order to define a new geometrical-tensional equilibrated state.

The forces at the anchorages are different from the ones previously found, and, therefore it is necessary to find again the new deformation state of the ring, which will be no more compatible with the last state of the net; for successive iterations, and, until the difference between the mutual actions between two successive state is less than a pre-fixed value, the final geometrical-tensional state, which will constitute the state 0 formed by the interacting ring-net set, is found.

prima di

+ piccolo

With the flow-chart in fig. 6a one can illustrate the research method of the cable-structure geometrical shape.

After having introduced the geometrical data, which consists of the initial co-ordinates of the anchorage points on the border ring and the forces-state in the cables owing to the pre-tension requested, one passes, appropriately checked by the parametrical data, to the equilibrium research of all the internal joints for successive iterations (parameter N STOP) and after having reached the equilibrium, one obtains an initial boundary condition.

dovute

richieste

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MANCA

After having checked the existence of the border structure, and after an affirmative answer, one passes to the calculation of the displacements in correspondence to the anchorage-joints, by means of a programme of space frame or finite element analysis.

The displacements, now obtained, modify the co-ordinates of the anchorage points beginning an iterative sequence (parame-

principio

ter NGBR), until convergence is reached, by means of control of the difference between two successive values of the displacements of the anchorage structure.

MANCA

6. - LOADING STATES

?

In order to follow with the calculation scheme, in the best possible way, the actual stress-strain behaviour of the structure in a time varying analysis, it has been decided to re-define a new state 0 ($\bar{0}$).

modo di funzionare

The new state $\bar{0}$ is defined considering as a true situation the combination of loading No. 4, corresponding to the presence of all the permanent loadings on the ring and cable-structure.

reale

Taking this situation as a base of calculation, one has considered the modification of the stress-state in function to infinite time of the reologic deformations of the perimetrical anchorage structure.

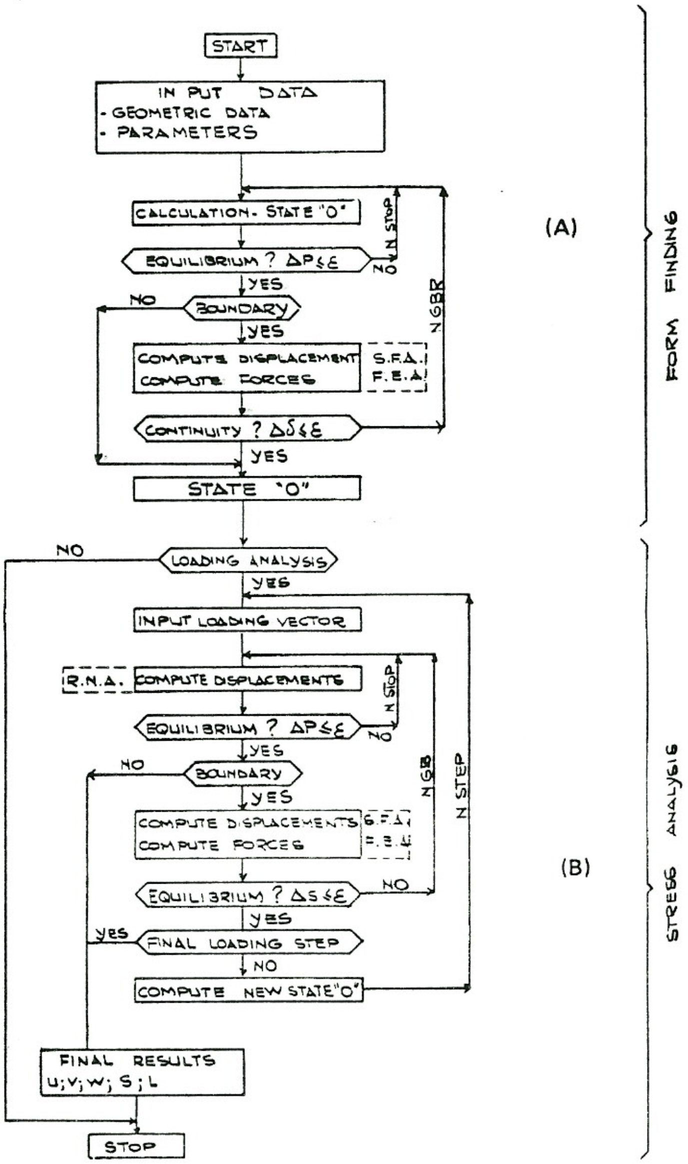
We have considered the condition of loading corresponding to the presence of the roof covering (this loading condition is present in the major part of the life of the structure) and, connecting us to the initial state 0 , we have considered, for the 4th combination of loading, the elastic modulus of the concrete at infinite time, with the relative losses of ring pre-stressing.

After having carried out the static calculation with these data, we have obtained as output a new geometrical-tensional state that will be useful to us, as a starting base for the valuation of the effects produced by accidental loading.

The above mentioned calculation has been carried out following the computer programme synthetized in the flow-chart of fig. 6b. Once the vector of loading has been introduced as mentioned above, the stress and structure deformation state of the cable structure is valuated, considering fixed boundary condition (R.N.A. = ROPE NET ANALYSIS). After checking that the cable net equilibrium is reached (NSTOP), the eventuality of the existence of the anchorage structure is taken into consideration.

Una volta

In the affirmative answer, which is our case, by means of a programme of space frame analysis (S.F.A.) or finite element analysis (F.E.A.) considered as subroutines of the main pro-



FLOW CHART — form finding and stress analysis

Fig. 6

gramme, the displacements of the ring structure in correspondence to the rope anchorages are calculated.

An iterative loop between R.N.A. and S.F.A. or F.E.A. is now beginning (NGB parameter) in a way to satisfy the compatibility of the displacements between the cable-structure and its anchorage structure.

In order to obtain an increased precision of the output calculations in function of the intensity of loadings, the possibility to act by incremental methods has been foreseen provveduto (N STEP), re-defining at every loading step the geometrical-tensional state. This calculation option has [illegible] allowed the definition of the new state $\bar{0}$ remarkably easily.
notevolmente facilmente

The variation of the sag (Δf) in the middle (point no. 154) of the cable structure and the displacement (δ) of a boundary point (no. 168), have been plotted in the diagram of fig. 7. From this diagram is possible to observe the convergence be-

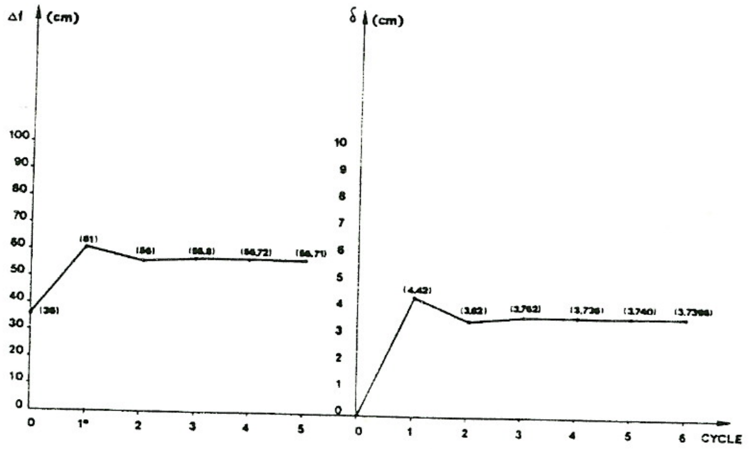


Fig. 7 – Iterative method convergence diagrams

haviour during the iterative sequence, for snow load condition, in order to obtain the necessary compatibility condition between ring and net structures.

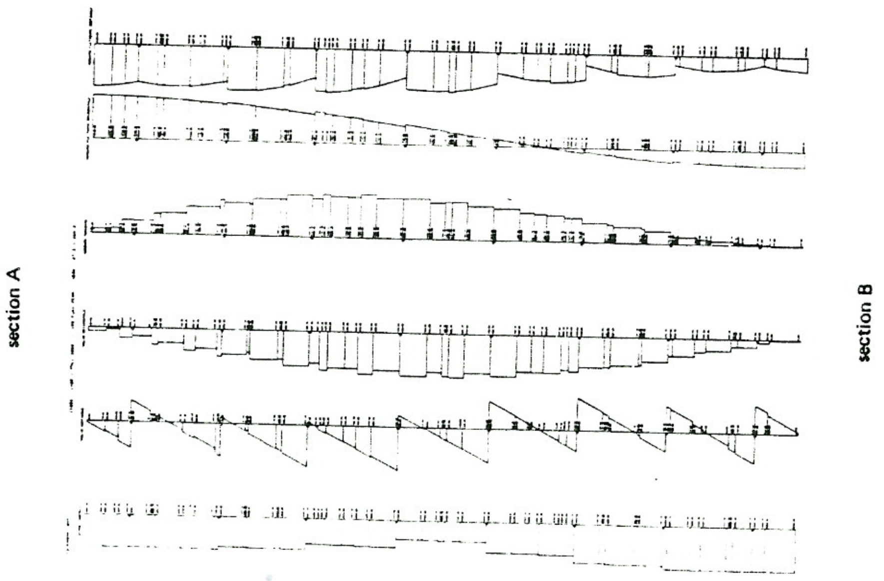
The interaction between the structures, due to the ovalization of the ring, can be taken into account with only 4 or 5 iterative cycles.

7. STATES OF MAXIMUM STRESS AND DEFORMATION (VERIFICATION)

The loading combinations no. 5 and no. 6 correspond respectively to to the loading of snow and wind action [illegible] on the covering. The ring has been designed in order to fit the best flexural distribution permitted by the bond represented by the displacement limitation of $\Delta f \leq 0.005$ L (L = maximum free span).

The maximum tensile force in the carrying rope for the snow load condition is of 121 TON; and 81 TON for the stabilizing rope during wind load condition.

According with all above mentioned, in fig. 8 is shown the



SNOW LOAD – automatic plotter of stresses in a quarter ring.

Fig. 8

taglio

final plotted diagram of axial forces, shear and moments for the snow condition (with the corresponding associated group of thermal variations in a quarter ring), obtained as output of the space frame analysis program.

In order to obtain more detailed information for the actual stress distribution the ring has been verified with a finite element analysis. In figs. 9 and 10 are shown the plan and axonometric view of the finite element mesh schematization for the ring actual structure. This structure has 6950 degrees of freedoms and took about 300 sec C.P.U. time of a C.D.C. CY 76 in order to examine 8 loading cases.

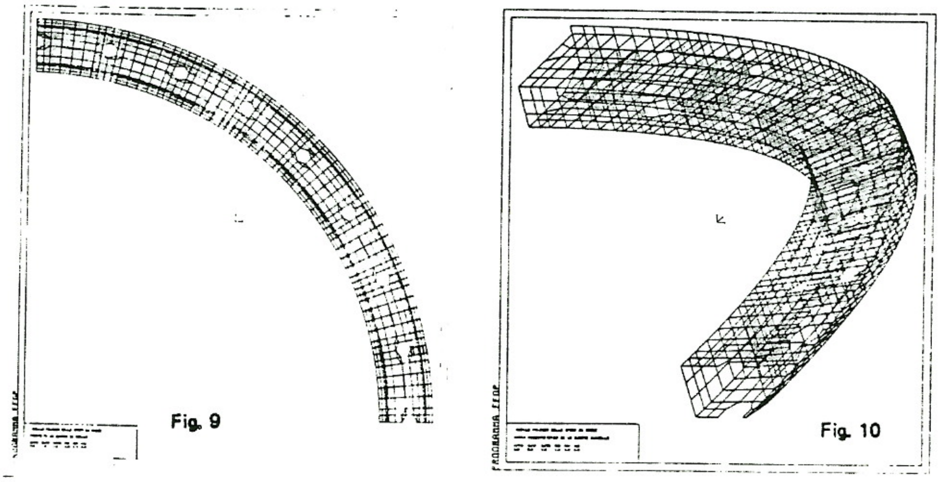


Fig. 9

Fig. 10

Finite element mesh schematization for the ring structure — Input geometrical data for computer programms.

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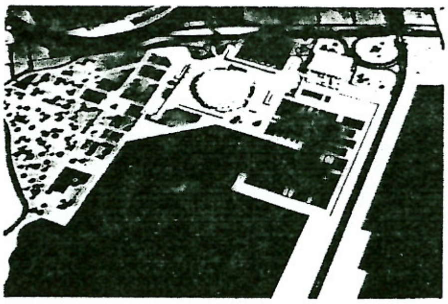


fig. 1

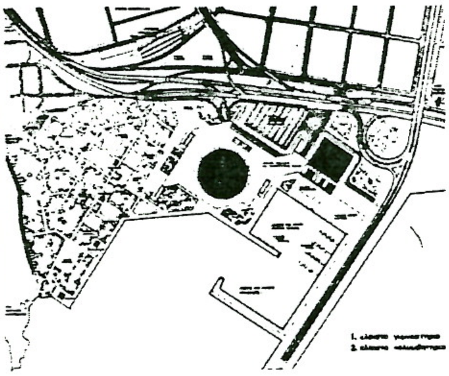
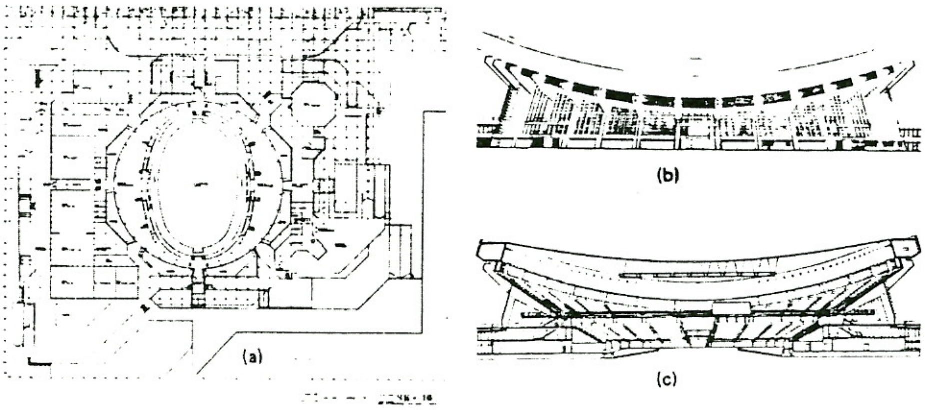


fig. 2



(a)

(b)

(c)

fig. 3

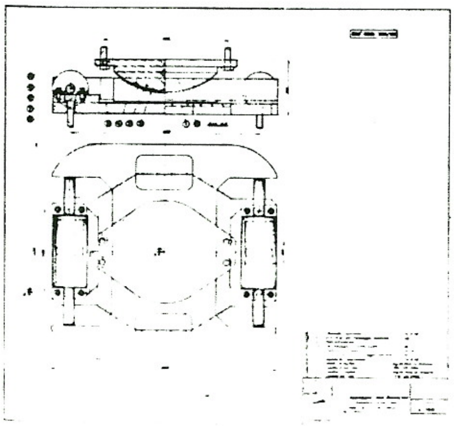


fig. 11 – Ring support