ACCIAIO 10/1976

GEOMETRICAL CONFIGURATION OF PNEUMATIC AND TENT STRUCTURES OBTAINED WITH INTERACTIVE COMPUTER AIDED DESIGN

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Abstract

In the current practice of planning of light spacial membranes (the weight of which is of about 1-3 kilos per sq. mt), several problems appear, which make this type of planning become very toilsome and expensive.

The first problem is represented by the research of the geometric shape that has to be given to the structural surface because the material which forms it, resisting only to traction stresses, can guarantee the stability and the resistance, further bearing in mind the fact that the geometry of the structural surface must also satisfy definite architectural requisites that have been previously fixed.

A preliminary process to fix the geometry of the structure is the one of the manufacture of models. But it is obvious that, as the attempts of project must be numerous, the manufacture of models would bring with itself remarkable and sometimes not acceptable times and costs. Further, this type of models (figure 1) can give a visual indication without giving any comfort for what refers to the tests of geometric idoneity, suitable to assure its static operation both in the condition of pre-stress and in the various steps of load.

The only convenient and quick method for the planning, both architectonic and structural of this type of realizations is offered by electronic programmes inter-actively structured. These programmes permit, by means of the use of a calculator, supplied with a display graphics system, a quick, visual and amusing design.

When the inter-active design has been finished, it is possible to obtain, by means of plottering, the visual geometric results on the screen: prospectives, prospects, as well as the necessary verifications.

In the first part of this work are reported the theoretic schematizations and the connections which rule the operation of the structure. In the second part, the automatic determination will be illustrated with numerous visualizations by means of the use of a "computer graphics display system".

IASS World Congress on Space Enclosures, Building Research Centre, Concordia University Montreal, July '76
Congres mondial de l'IASS sur les enceintes spatiales. Centre de recherches sur le bâtiment, Universite Concordia, Montréal, juillet '76

1. GEOMETRY AS CONSEQUENCE OF THE STATE OF STRESS

1.1. Membranes with stiff contour: Connections between state of stress and geometry of membranes

As it is known, the indefinite equations expressing the conditions of balance to the translation of an infinitesimal element of membrane, according to the three directions x, y, z forming a generical Cartesian reference, can be written in the following way:

$$(1)\quad (\sum X=0)\qquad \frac{\partial \bar{n}_x}{\partial x}+\frac{\partial t}{\partial y}+\bar{p}_x=0$$

$$(2)\quad (\sum Y=0)\qquad \frac{\partial \bar{n}_y}{\partial y}+\frac{\partial t}{\partial y}+\bar{p}_y=0$$

$$(3)\quad (\sum Z=0)\qquad \bar{n}_x\frac{\partial^2 z}{\partial x^2}+\bar{n}_y\frac{\partial^2 z}{\partial y^2}+2t\frac{\partial^2 z}{\partial x\,\partial y}= -(\bar{p}_z+\bar{p}_x\frac{\partial z}{\partial x}+\bar{p}_y\frac{\partial z}{\partial y})$$

in which, with the usual meaning of symbols:

$\bar{n}_x$ = projection on the plane $Z=0$ of the normal stress according to x,

$\bar{n}_y$ = projection on the plane $Z=0$ of the normal stress according to y,

$\bar{t}_{xy}=\bar{t}_{yx}=\bar{t}$ = projection of the shearing stress on the plane $Z=0$,

$\bar{p}_x$, $\bar{p}_y$, $\bar{p}_z$ = components of p for unit of surface projected on the planes yz, xz, xy.

The connections among the projectes stresses and the membrane stresses are:

$$(4)\qquad \bar{n}_x=n_x\frac{\sqrt{1+(\frac{\partial z}{\partial y})^2}}{\sqrt{1+(\frac{\partial z}{\partial x})^2}}$$

$$(5)\qquad \bar{n}_y=n_y\frac{\sqrt{1+(\frac{\partial z}{\partial x})^2}}{\sqrt{1+(\frac{\partial z}{\partial y})^2}}$$

$$(6)\qquad \bar{t}=t$$

Considering the expressions (1), (2) and (3) it is possible to observe that, in substance, there are two ways suitable to solve the problem of project.

a) The surface $z=f(xy)$ is known, and it desired to obtain the state of stress.
This type of process requires that the geometry of the structure can be considered as unchangeable.
Without making considerations on the stability of the balance, in this case the problem is centered in the verification of the state of stress of the membrane, considering that the shape of the structure and the conditions of load authorize to neglect the flexional actions.
When the structure taken in examination is composed by material resisting only to traction (tensostructures, pneumatic structures, etc.), the given surface should also allow to verify the condition

$$(7)\quad n_{min}=\frac{1}{2}\,(n_x+n_y\pm\sqrt{(n_x-n_y)^2+4t^2})\geq 0$$

It is obvious that only in very simple cases it is possible to find surfaces that can easily be analitically expressed which satisfy the (7) (for example, the sphere is a perfect pneumatic surface). In the case of arbitrary boundary conditions, and lacking of radial symmetry, it is almost impossible to define a surface compatible with the (7). In the figure 2 is illustrated an interactive sequence of a function $z=f(xy)$ with double opposite curvature, suitable to satisfy the (7). In the figure 3 has been visualized an attempt for the research of a surface suitable to satisfy the (7) in the presence of internal pressure.

b) The regimen of stresses is known, and the balance equations are requested to supply us with the geometry of the balanced surface.

In the cases that frequently recur in the practice of the designer, the equation (3) obviously bound to the (1), (2), appears, considering as unknown function the $z=f(x,y)$, as an almost linear differential equation to the derivatives of the second order having variable coefficients.

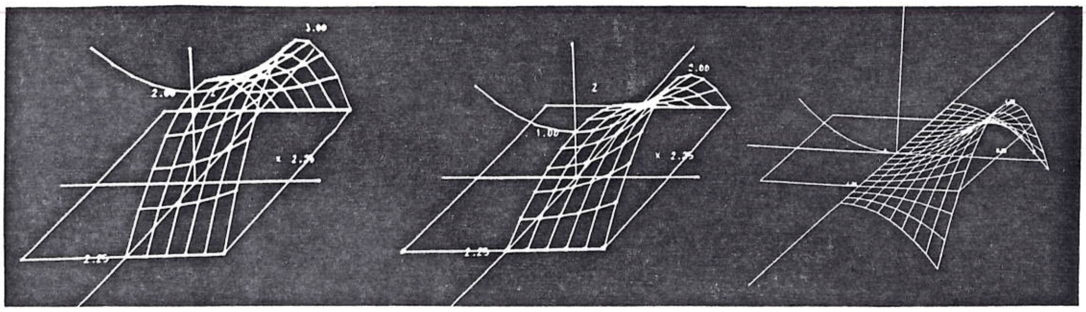

Fig. 2

The equation (3) to the partial derivatives can therefore be classified as hyperbolic, parabolic or elliptical, according to the fact that its discriminant $(t^2 - \bar{n}_x \bar{n}_y)$ is greater, equal, or smaller than zero.

Noting instead that the sign of the discriminant $(t^2 - \bar{n}_x \bar{n}_y)$ is invariable with reference to whatever transformation of actual continuous co-ordinates and differentiable with jacobian $\neq 0$, we can pass to a system of curvilinear co-ordinates coinciding with the main directions of the stresses $(t = 0)$.

In this reference we therefore have that the differential equation to the partial derivatives is

hyperbolic if $n_x n_y < 0$

parabolic if $n_x n_y = 0$

elliptic if $n_x n_y > 0$

From the previous classification it si possible to draw that for the structures with hyperbolic surface $(n_x n_y < 0)$ we shall have that the stresses will be of opposite sign; for the parabolic surfaces in one of the main directions we shall have e nil stress (cones and cylinders), and that for the elliptic surface the stresses are of the same sign. These conditions on the coefficients of the differential equation define a well determined regimen of internal stresses for the generical unknown function $z = f(x,y)$. In our case, having fixed beforehand the value and the sign of the coefficients t, n_x, n_y (which define the state of internal stress of the membrane) we can foresee the type of unknown surface which can be classified as hyperbolic, parabolic or elliptic by the variable coefficients t, n_x and n_y.

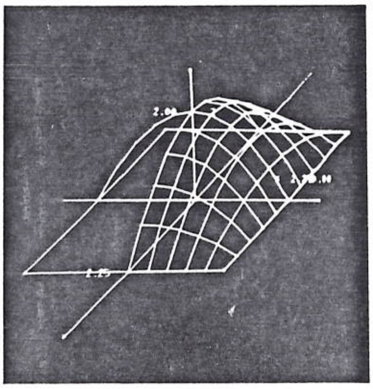

Fig. 3

Operating on these coefficients, we can define the most convenient structural surface for the constructive reality for which it is foreseen.

From the geometric point of view, it is important to notice that, considering $K = k_1 k_2$, the Gaussian curvature, the structural surface is:

hyperbolic if $k_1 k_2 < 0$

parabolic if $k_1 k_2 = 0$

elliptic if $k_1 k_2 > 0$

1.2. Different forms taken by the equation (3) in special conditions of load and of regimen of stress

a) Pre-stressed membrane with nil external loads

In this case the equation (3) takes the form:

(8) $$\bar{n}_x \frac{\partial^2 z}{\partial x^2} + \bar{n}_y \frac{\partial^2 z}{\partial y^2} + 2\, t \frac{\partial^2 z}{\partial x\, \partial y} = 0$$

and in the hypothesis of considering $\bar{n}_x = \bar{n}_y$ and $t = 0$ it is obtained:

(9) $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

an equation which show the hyperbolic specification $(k_1 k_2 < 0)$ of the geometric surface solution of the (9), caused by the forced elliptic regimen of the stresses $(\bar{n}_x \bar{n}_y > 0)$.

The introduction of a state of internal coaction, for example of pre-stressing, can change the regimen of stresses from elliptic to hyperbolic $(n_x n_y < 0)$ in a still geometrically hyperbolic structure $(K < 0)$. This is the typical case of the tensostructures in which a pre-stress of traction is introduced assuring the stability of the structure itself.

b) Membrane undergoing the action of its own weight and of vertical incidental pressures

The equation (3) becomes, considering the direction z as vertical,

(10) $$\bar{n}_x \frac{\partial^2 z}{\partial x^2} + \bar{n}_y \frac{\partial^2 z}{\partial y^2} + 2\, t \frac{\partial^2 z}{\partial x\, \partial y} =$$

$$= - p_p - p_z \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$$

in which: p_p = own weight of the membrane,
p_z = incidental loads with direction z.

c) Minimum surfaces

A surface with minimum area is a surface for which the main curvature (H) is nil in all its points.

Remembering that in a Cartesian reference (x,y,z) the expression of the main curvature is:

(11) $$H - \frac{\left[1+\left(\frac{\partial z}{\partial y}\right)^2\right] \frac{\partial^2 z}{\partial x^2} + \left[1+\left(\frac{\partial z}{\partial x}\right)^2\right] \frac{\partial^2 z}{\partial y^2}}{2 \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}^{\,3}} +$$

$$- 2 \frac{\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \frac{\partial^2 z}{\partial x\, \partial y}}{2 \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}^{\,3}}$$

and that the equation of membrane for uniform state of stress (n_o) undergoing to the action of its own weight, external action shared and with internal pressure (pneumatic structures) is given by the:

(12) $$2(n_o + p_p z)H = \pm \left[p + \gamma(z_o - z)\right] +$$

$$- p_p \frac{1}{\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}}$$

γ = specific weight of the liquid

we obtain that for $p_p = \gamma = 0$, the curvature acquires the constant value of $H = \pm\, p/(2\, n_o)$ and that the surface results minimum also if it is $p = 0$.

Therefore a surface with minimum area can be obtained in absence of loads and in a uniform state of stress.

In the figure 4 is visualized the minimum surface on a contour of six sides.

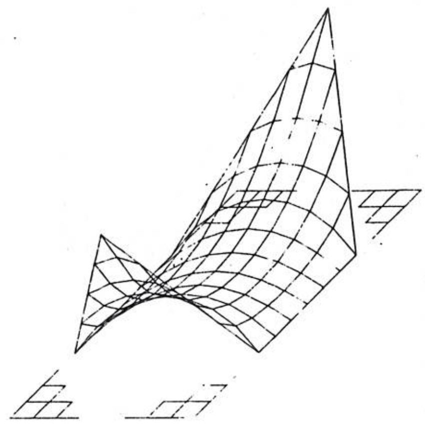

Fig. 4

d) Membrane structures undergoing to internal pressure

The equation "of membrane", which permits to find the balanced geometric configuration of the structure, appears in the following form:

$$(13)\quad \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \left(\frac{\partial z}{\partial x}\right)^2 \frac{\partial^2 z}{\partial y^2} - 2\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial^2 z}{\partial x \partial y}\right) + \left(\frac{\partial z}{\partial y}\right)^2 \frac{\partial^2 z}{\partial x^2} = \frac{p}{n_o}\left(\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\right)^3$$

In the case that the own weight of the membrane is taken into consideration, the term at the right of the equal changes and becomes:

$$(14)\quad 1 + \frac{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}{\frac{n_o}{P_p} + z}$$

In the figure 5 is represented the surface of the membrane undergoing to internal pressure ($P/n = 0.2$).

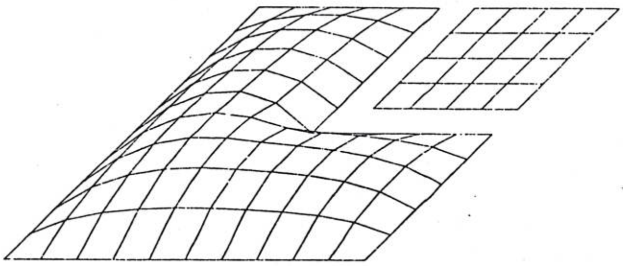

Fig. 5

1.3. The numerical solution

The equation (3) and all its transformations can be written at the finite differences. In this way, instead of solving a differential equation, the approximate solution is supplied by a system of equations, in general not linear.

The expression (8), written for the generical internal way (i;j) to the finite differences, appears in the following way:

$$(15)\quad z_{i;j}\left\{4 + \frac{1}{2\,h^2}\left(z_{i+1} - z_{i-1;j}\right)^2 + \frac{1}{2\,h^2}\left(z_{i;j+1} - z_{i;j-1}\right)^2\right\} =$$

$$= z_{i;j-1} + z_{i+1;j} + z_{i;j-1} + z_{i;j+1} + \frac{1}{4\,h^2}\left(z_{i+1;j} - z_{i-1;j}\right)^2\left(z_{i;j-1} + z_{i;j+1}\right) + \frac{1}{4\,h^2}\left(z_{i,j+1} - z_{i;j-1}\right)^2\left(z_{i;j-1} + z_{i;j+1}\right) - \frac{1}{8\,h^2}\left(z_{i+1;j} - z_{i-1;j}\right)\left(z_{i;j+1} - z_{i;j-1}\right) \cdot \left(z_{j+1;i-1} + z_{j-1;i+1} - z_{j-1;i-1} - z_{j+1;i+1}\right)$$

The boundary conditions are instead expressed, generally, by giving the co-ordinates of the external points of edge, considered to be fixed.

Other types of conditions at the contour can be expressed, bearing in consideration the hyperbolic or elliptic characteristics of the differential equation.

For what refers to the iterative method, chosen for the solution of the system of equations composed by the (15) written for all the internal nodes, the Gauss-Seidel method has been adopted for the expression which rules the operation of the pre-stressed membranes in absence of loads and for the minimum surfaces. In the case of membranes undergoing loads, as the pneumatic structures, the Gauss method supplies in general a weak convergence.

And then it is advisable to solve this type of structures by means of the method reported in the section 2.

2. MEMBRANES WITH MOBILE CONTOURS

In the case that the membrane is supplied with an edge, composed only by anchorage points, the equations (1), (2) and (3) cannot in general be used any more for the purpose of obtaining a solution to the problem of the research of the geometric configuration. In this case it is convenient to start considering a net made of nodes and of fictitious rods, as well as to require in the space the balance of the net that will thus be formed.

The balance of the generical node k, to which flows a number m_k of rods, in the step 0 in which the geometry is supposed to be known, is scalarly expressed by the

$$(16)\quad \begin{cases} \sum_{1}^{m_k}{}_{ki} \frac{S^{\circ}_{ki}}{L^{\circ}_{ki}} \Delta x^{\circ}_{ki} = P^{\circ}_{xk} \\ \sum_{1}^{m_k}{}_{ki} \frac{S^{\circ}_{ki}}{L^{\circ}_{ki}} \Delta y^{\circ}_{ki} = P^{\circ}_{yk} \\ \sum_{1}^{m_k}{}_{ki} \frac{S^{\circ}_{ki}}{L^{\circ}_{ki}} \Delta z^{\circ}_{ki} = P^{\circ}_{zk} \end{cases}$$

in which:

$$\Delta x^{\circ}_{ki} = x^{\circ}_k - x^{\circ}_i , \quad \Delta y^{\circ}_{ki} = y^{\circ}_k - y^{\circ}_i , \quad \Delta z^{\circ}_{ki} = z^{\circ}_k - z^{\circ}_i$$

$$L^{\circ}_{ki} = \sqrt{(\Delta x^{\circ}_{ki})^2 + (\Delta y^{\circ}_{ki})^2 + (\Delta z^{\circ}_{ki})^2}$$

$x^{\circ}_k, y^{\circ}_k, z^{\circ}_k$ = co-ordinates of the generical internal point k in the step 0,

S°_{ki} = stresses in the rod ki in the step 0,

L°_{ki} = length of the rod ki in the step 0,

$P^{\circ}_{xk}, P^{\circ}_{yk}, P^{\circ}_{zk}$ = components of the load in the step 0 according to the directions x, y and z respectively.

The connections (16) are written for each internal node, thus giving us $3n$ equations, being n the number of "free" internal nodes. In a compact way the system of equations can be written:

$$(17)\quad [A]\{S\} = \{P\}$$

in which S is the vector of stresses, A is the matrix of cosines directors of the elements (rods) and P is the vector of loads.

The global unknowns of the problem are the m_k values of the stresses S_{ki} in the rods and the $3n$ co-ordinates of the internal nodes.

The solution, in a practical order, of our system of equations of balance, can be faced in various ways, according to the quantities chosen as unknown ones.

We can thus follow two ways:

a) in the case that the geometry of the structural surface is not known,

b) in the case that the geometry of the structural surface is known.

a) Geometry of the unknown structure

The equations (16) can be solved, when the stresses S_{ki} have been fixed on the rods, requesting from a system of $3n$ not linear equations the $3n$ unknown co-ordinates of the nodes.

This type of solution is advisable for the research of the initial geometric configuration, when the conditions at the contour are free, but it meets difficulties in the numerical solution, owing to the not linearity, frequently remarkable, of the system to be solved.

For the purpose to make easier the convergence of the iterative process (Newton-Raphson), a first value of the co-ordinates of nodes can be obtained by giving arbitrary values to the ratio $S_{ki}/L_{ki} = q_{ki}$. In this way a linear system of equations is solved obtaining the values of the first iteration with the possibility of obtaining a remarkable acceleration of the convergence if the given initial values are suitable.

The iterative method used to solve the equations of balance continues than in the known way: obtained the first result with the solution of the linearized system:

$$[\Delta]\{q\} = \{P\} ,$$

in which with $[\Delta]$ has been shown the matrix formed by the differences Δx°_{ki} ; Δy°_{ki} ; Δz°_{ki} and with $q_{ki} = S_{ki}/L_{ki}$ the ratio between force and length of the rod $\bar{k}_i$, we pass to the iterative step with at disposal the first set of solutions (x°; y°; z°; S°; L°).

The further iterations are obtained with the method of the successive substitutions. With this method the vector of loads is increased step by step and the not linear steps are up-to-dated at each iteration. In a generical (r) iteration we shall have:

$$[A]^r \{S\}^r - \{P\} = \{\Delta P\}^r$$

The iterative sequency is continued until the out-of-balance $\{\Delta P\}^r < \mathcal{E}$.

In the current practice are frequent bonds of project in the definition of the net formed by the rods. The most frequent restrictive conditions are the ones of imposing stresses or constant lengths in a whole of rods.

In a specific case of a net of cables it means to have cables sized at the optimum of resistance in the case of prescription of constant stresses along a line. And of easier pre-fabrication and ease of assembling in the case of constant lengths.

If, after having obtained the desired geometry and state of stress, it is required to introduce restrictive conditions on stresses or on lengths of rods, changing the minimum possible the starting geometry itself, an advantageous method is the one to find the compromise solution by means of not linear programmation.

Considering to introduce $r < m$ conditions on the lengths and on the stresses of r rods, the problem of not linear programmation can be introduced as follows:

$$\begin{cases} \text{variables: } [x,\ y,\ z,\ S] \\ \text{objective function: } f = \sum \left[(x - x_o)^2 + \right. \\ \qquad \left. + (y - y_o)^2 + (z - z_o)^2 \right] \rightarrow \min \\ \text{bonds: } g(x,\ y,\ z,\ S) : 0;\ S > 0 \end{cases}$$

in which x, y, z co-ordinates of nodes (unknown)

x_o, y_o, z_o co-ordinates of nodes in the known initial condition

S stress in the rods

The objective function and the bonds are not linear with reference to the variables, and therefore the solution is found by means of a sequence of minimization with the suitable choice of the function of penalization.

The bonds are represented by the 3n equations of balance and by the r conditions given to the stresses and to the lengths of rods.

b) Geometry of the known structure

In this case are known the 3 n co-ordinates of nodes. Projectually speaking, this means that the structural geometry has been defined for reasons of architectonic order. Usually a thus defined surface does not satisfy the conditions of balance. The solution of the problem will consist in finding the nearest balanced surface to the one of project.

Known therefore the matrix $[A]$ in the (17) we have as unknown the vector $\{S\}$ of stresses in the rods. In this way we arrive to a system of 3 n equations with m unknowns with $3n > m$.

This system, generally, does not admit an univocal solution, unless we have $3n - m$ linearly depending equations and therefore the degree of the matrix $[A]$ of order m .

Also of this process we limit ourselves, for the time being, to a hint. We are now looking for, as written in (a), a solution of first attempt, thus obtaining a first set of values for stresses in the rods contained in the vector $\{S\}_o$. The vector $\{S\}_o$ will not be able however to satisfy the conditions of balance associated to the matrix $[A]$ known of cosines directing, and therefore will exist a lack of balance $\{\Delta P\}$ in nodes where

$$(18) \qquad \{\Delta P\} = [A]\{S\} - \{P\} = 0$$

The value of the vector of stresses $\{S\}$ which, associated to the matrix $[A]$, will numerically satisfy the balance, can be obtained minimizing the difference $\{\Delta P\}$ with the method of least squares. Obtained the value of $\{S\}$ which minimizes $\{\Delta P\}\{\Delta P\}'$ the final geometry of the surface can be obtained by means of the method reported in (a).

It is possible to reach directly the same result by means of the following formulation in terms of mathematic programmation:

$$\begin{cases} \text{variable: } [x, y, z, S] \\ \text{objective function: } \sum \left[(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 \right] \rightarrow \min \\ \text{bonds: } g(x, y, z, S) = 0; S > 0 \end{cases}$$

The objective function represents the basic request to satisfy the nearest possible the starting geometry.

The bonds are represented by the balance equations.

2.2. Numerical results

The figures 6 and 7 represent the position taken by the net of rods with rest in four positions, having requested respectively constant length and constant efforts in the internal rods.

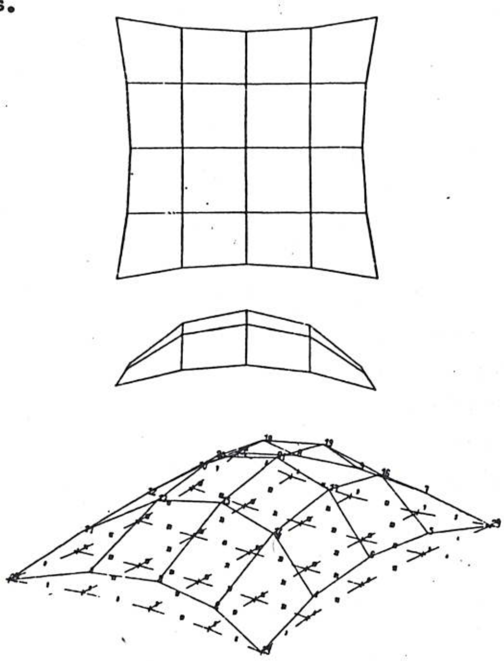

Fig. 6

The figure 8 shows the position of balance of a sail anchored on three points, undergoing to a constantly direct load.

A structure with a remarkable number of nodes and rods, anchored in 7 external points at different levels and with internal bearing, is visualized in axonometry (fig. 9).

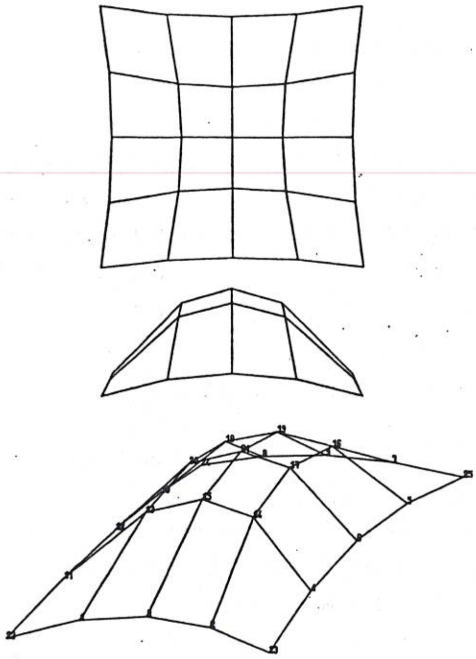

Fig. 7

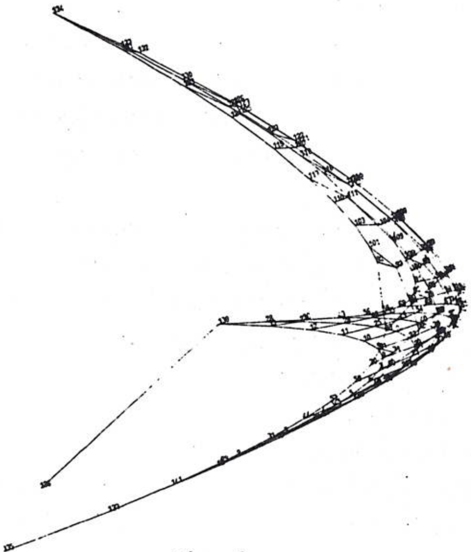

Fig. 8

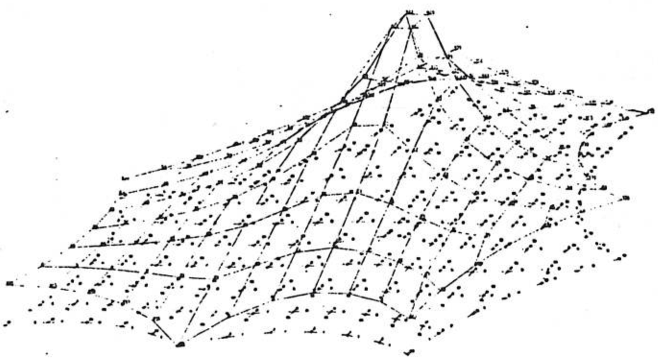

Fig. 9

3. THE INTERACTIVE PLANNING COMPUTER-ENGINEER

In the research of the surface of membranes it is of remarkable assistance for the planning to obtain the plotterized outputs from the computer, thus making immediately visible the result of the made calculations.

An even greater advantage can be obtained by means of the use of a video-display which permits to visualize instantly, by means of a cinescope, what a programme sends in output. The advantage of such a device is remarkable if the structural organization of the programme can avail itself of a suitable bi-directional organization (that is if it can receive information and supply results).

An electronic programme, "bi-directionally structured", during the performance of the operation which elaborate the known values, can momentarily interrupt the calculation, make modifications to data, and start again, elaborating the new values.

In order that such a programme can operate, it has to be capable of being introduced into an inter-active computer, that is such to allow to operate from the outside, acting on it when it is deemed advisable, according to a typical operation of correction, attempt, or verification of project.

The computers supplied with these programmes are not any more instruments, but they have the possibility of being inter-active in a designing sense, thus becoming helps to think and to solve a problem of engineering or of architecture.

The video-display thus becomes the instrumental and mental extension of the operator, increasing in an enormous way the capacity, quality of designing and offering, further to this, the possibility of easily the optimal solution.

With the diagram with blocks of the figure 10 it has been shown in scheme the inter-active operation of the programme of calculations.

To illustrate an inter-active sequency, in the figure 11 has been visualized on the video a type of structure, modified according to the exigencies of the project.

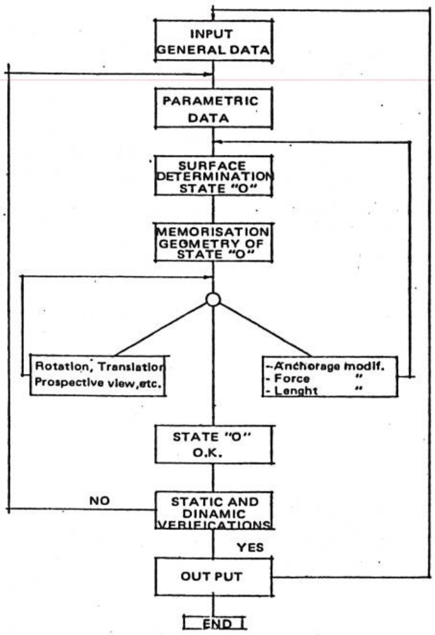

Fig. 10

The sequency illustrates the research of the surface in balance, anchored to dis-connected points at different heights.

In the sequencies are visualized some inter-active steps of the calculation, the final choice of the geometric shape with possible rotations and the plotterizations on paper, obtained after that the requested requisites had been satisfied. On plotter also the graphic indication of the flat strip canvas pattern which, assembled together, will form the spacial surface which cannot be developed of the structure (figure 12) has been obtained.

Figure 13 shows the final result of finding the pneumatic structure anchored in a square of 36 x 36 mt.

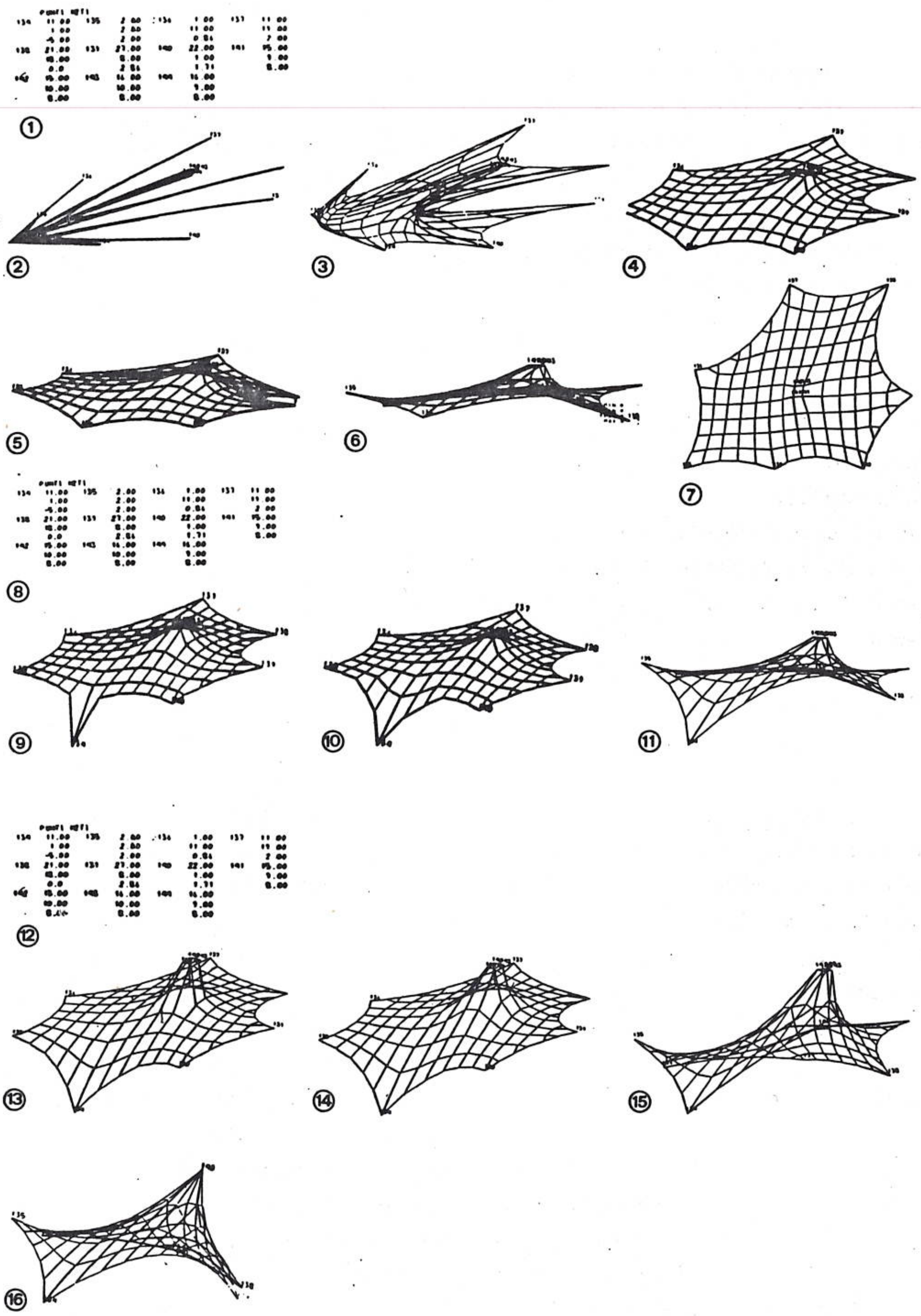

Fig. 11 - Interactive sequence of project with the video display Consol IBM 2250.

(1) Coordinates of anchorage points; (2), (3) Visualization of the iterative calculation; (4) convergence achived; (5), (6) rotation; (7) Plant view; (8), (9) changement of an anchorage point; (10) finding the new equilibrated surface; (11) rotation; (12), (13) changement in the peak points; (14) new equilibrated shape; (15) rotation; (16) the four internal peak points become a single point plus rotation.

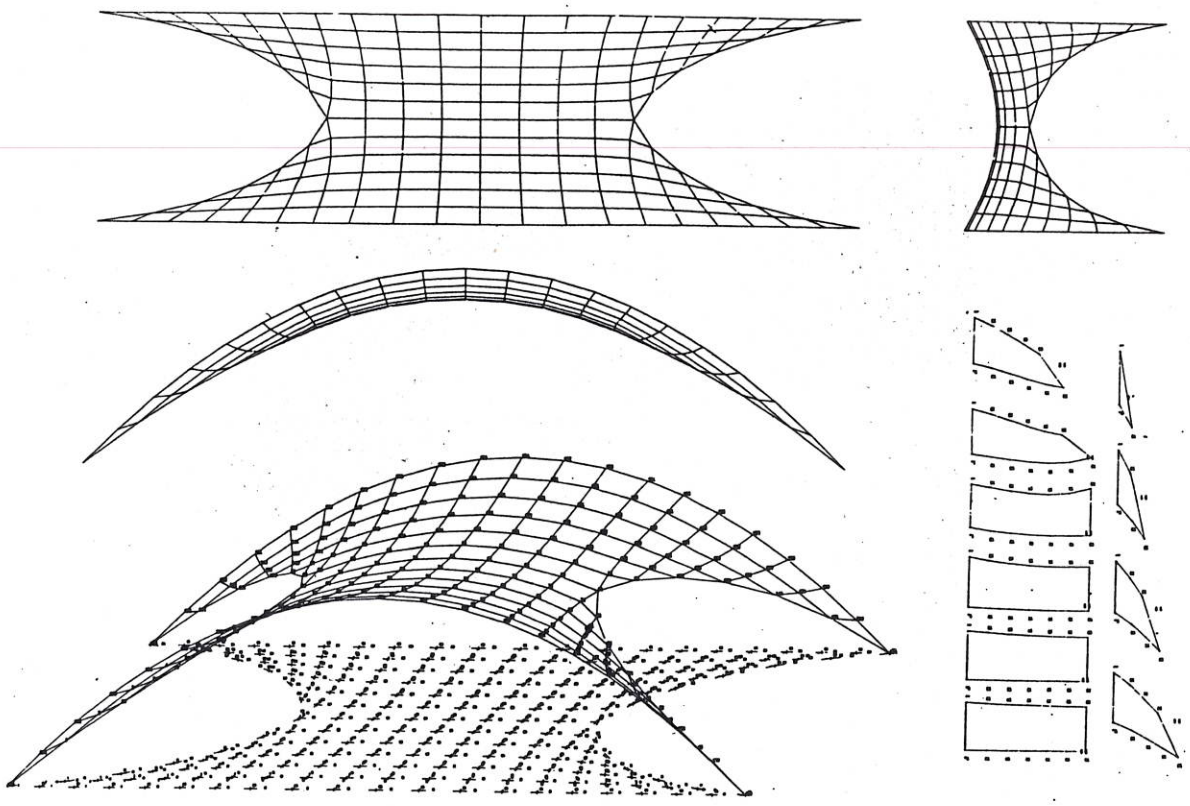

Fig. 12

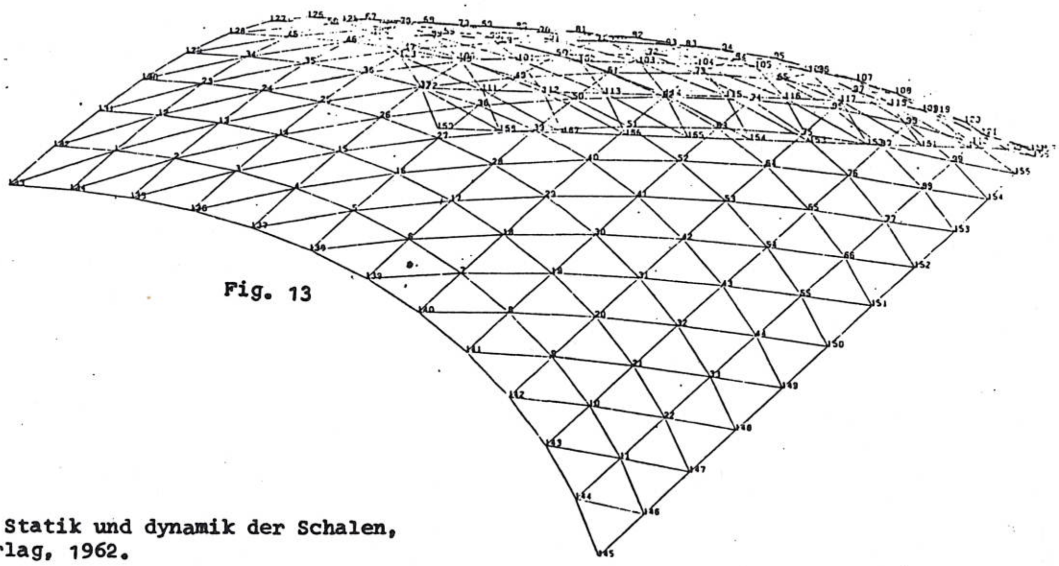

Fig. 13

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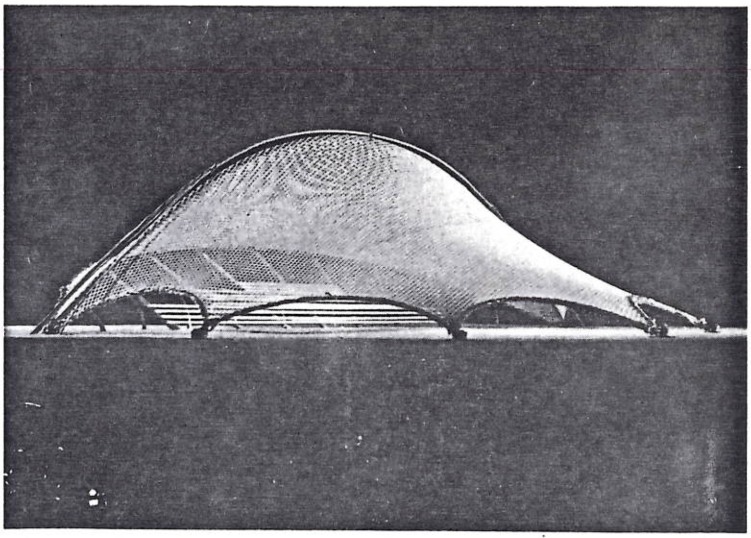

Fig. 1