Large-span Pneumatic Structures, Reinforced by Steel Cables

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Introduction

An international conference on pneumatic structures was held in Venice from the 13th to the 18th June, 1977, attended by leading specialists in this field. The authors of this article had the opportunity of displaying some of the most representative buildings of recent years.

In the first part of the article, a short summary is given of a theoretical method for designing pneumatic structures. These designs are drawn up with the aid of interactive computers, such as the Tektronik 4010 visual display unit. Calculation programs (pneuss), themselves also interactive, were developed to allow automatic preparation of the data (prepne) and of the graphic "outputs" (pneuss plot).

The second part of the article comprises the description of various buildings already completed, which have considerable importance on a world-wide scale, for example the largest pneumatic roof in the whole world, at Detroit (U.S.A.).

A. Search for the Balanced Geometric Configuration of a Pneumatic Structure under Pressure

A.1. *Outline of the Problem*

The equations of equilibrium, in terms of the membrane, of an infinitely small surface element, present drawbacks with regard to convergence, and difficulties concerning the formulation of edge conditions of the continuous model. In order to avoid these difficulties, it is advisable to have recourse to a mathematical model, made of flat elements of scaled-down surface area, which are assembled to form the pneumatic surface in equilibrium.

A.2. *The Surface Element*

Let us consider a typical node $K = (X_k; Y_k; Z_k)$ belonging to the surface of equilibrium and let us examine the triangular elements which have their vertex at K (Fig. 1).

From these elements, we isolate one (1, 2, 3) (Fig. 2) in relation to the axes of local co-ordinates. It is assumed that the state of stress in this element is constant.

By considering the forces F_1, F_2, F_3, statically equivalent to the zone of constant tension and controlled according to the sides of the triangle, we have:—

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T] \cdot \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \qquad (1)$$

where

$$[T] = \frac{2}{t} \begin{Bmatrix} \dfrac{l^2_{12}}{h_3} & \dfrac{l^2_{23}}{h_1} & \dfrac{l^2_{31}}{h_2} \\ \dfrac{m^2_{12}}{h_3} & \dfrac{m^2_{23}}{h_1} & \dfrac{m^2_{31}}{h_2} \\ \dfrac{l_{12}m_{12}}{h_3} & \dfrac{l_{23}m_{23}}{h_1} & \dfrac{l_{31}m_{31}}{h_2} \end{Bmatrix} \qquad (2)$$

t = thickness of the membrane;

h_1, h_2, h_3 = the altitudes of the triangle, measured respectively from the vertices 1, 2 and 3;

l_{kj} and m_{kj} = the director cosines of the side KJ.

The fact of supposing that the stresses in the element are constant makes it possible to consider the node K as subjected to loads in equivalent imaginary bars which converge at this node.

One may therefore consider, instead of the actual continuous structure, a unit made of imaginary nodes and bars, especially at the stage of finding the surface of equilibrium, where, by the very nature of the method, expansion of the structure is not involved. The imaginary equivalent structure obtained by this method is therefore a reticular surface formed of bars which are the adjacent sides of the small triangles.

This method makes it possible, after having found the loads F, to return by means of the equations (1) to determining the main stresses in the continuous material which constitutes the supporting envelope.

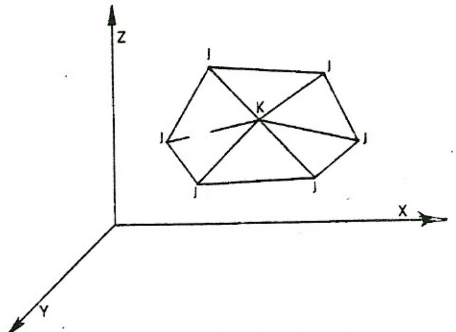

Fig. 1. The pneumatic structure is broken up into triangular elements. K is any node of co-ordinates X_k, Y_k, Z_k.

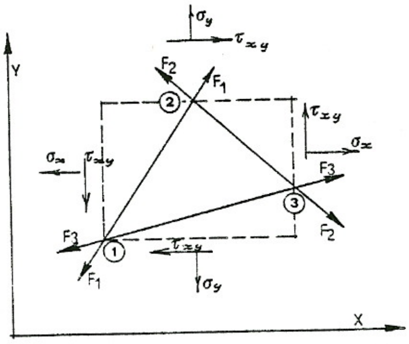

Fig. 2. Triangular element (1), (2), (3) subjected to loads directed along the three sides.

A.3. *Overall Equilibrium*

By assembling all the elements, the equation of equilibrium which is to solve our problem is given vectorially by:—

$$\sum_{j=1}^{NA} \overline{S}_{kj} = p \sum_{j=1}^{NA} \overline{n} A_{kj} \qquad K = 1, N \qquad (3)$$

where

N = number of internal nodes;

NA = number of bars converging at the node K;

p = internal pressure;

$\overline{S}_{kj}$ = equivalent loads directed along the sides of the elements and formed of the sum of the forces F in two adjacent sides;

$\overline{n}$ = vector of the normal to the surface of an element having its vertex at K;
A_{kj} = area of the element having its vertex at K.

Expressed in a compact form, and related to the system of general co-ordinates, the equation becomes:—

$$[\overline{T}]\ \{S\} = \{P\} \qquad (4)$$

where

$[\overline{T}]$ = matrix of the cosines of the angles formed by the bars with the axes of co-ordinates.

A.4. *Numerical Solution*

Given the values of the loads in the imaginary bars, one may solve a system of non-linear equations, where the unknown quantities are the values of the co-ordinates of the internal nodes of the structure.

Let us consider, therefore, that we have in this manner obtained the numerical values of the following quantities:—

$P^0_{kj} \equiv (X_k;\ Y_k;\ Z_k)$ co-ordinates of the internal nodes);

$S^0_{kj} \equiv$ load in the imaginary bar kj;

$L^0_{kj} \equiv$ length of this bar.

This first result cannot be considered satisfactory, especially for a pneumatic structure. In fact, if the latter has a rectangular contour, the altitudes obtained near the corners are very small and, for this reason, the solution lacks practical interest.

In order to avoid these drawbacks, it is necessary, complementary to the conditions of equilibrium, to introduce other conditions or "restraints" to be fixed for each design.

The problem to be solved appears as follows:—

(*a*) It is necessary to find a new surface of equilibrium and for that new values of $P^F_k = (X^F_k,\ Y^F_k,\ Z^F_k)$;

(*b*) The surface will be subjected to "restraints", such as:—

- Predetermined values of the L^F_{ki} (non-linear restraint);
- Predetermined values of the S^F_{kj} (linear restraint);
- Predetermined values of X^F_k, Y^F_k, Z^F_k (linear restraint);

(*c*) The choice between the infinite number of surfaces which may be obtained with such restraints will be conditioned by an objective function. The latter, generally non-linear, must enable us to find a surface as near as possible, taking the restraints into account, to the surface desired by the designer.

Hence the idea of solving the problem (finding the surface to be chosen finally) by an automatic optimisation method.

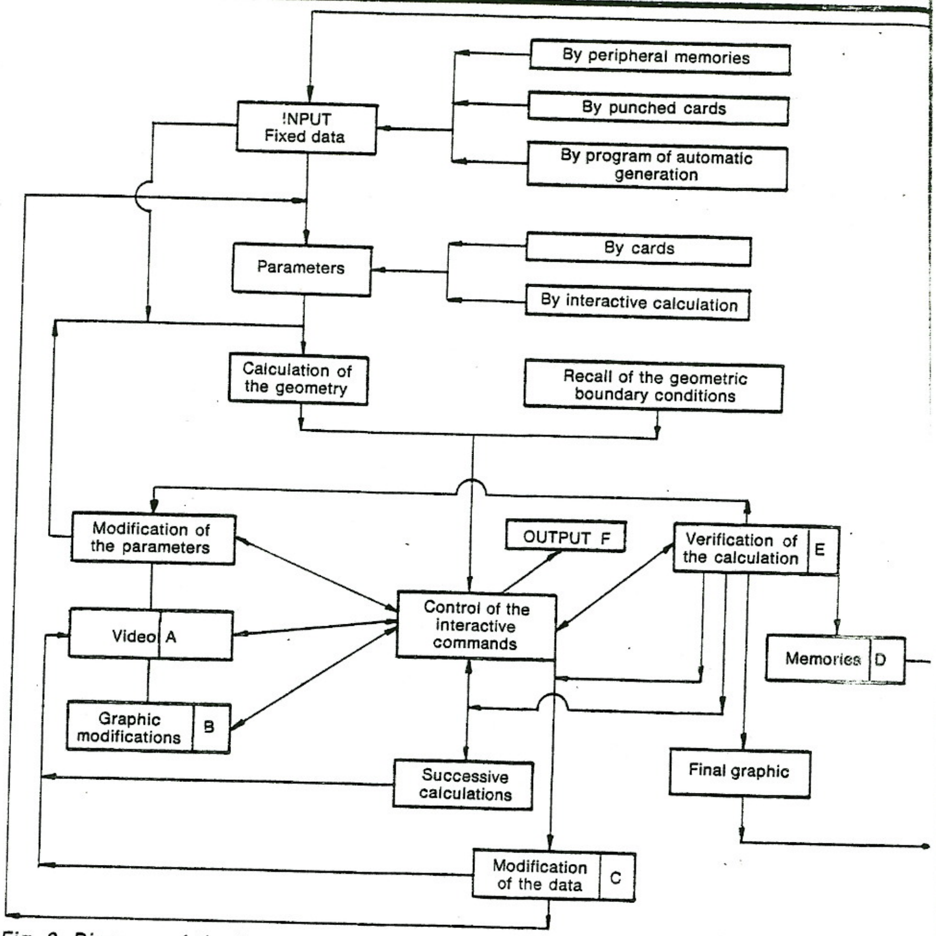

Fig. 3. Diagram of the interactive calculation of a pneumatic structure.

Thus one has:—

- Variables: $(X^F,\ Y^F,\ Z^F,\ S^F)$;
- Restraints: $g\ (X^F,\ Y^F,\ Z^F,\ S^F) = 0$ with $S > 0$;
- An objective function:—

$$F = \sum_{1}^{n}{}_k\, C_1\ [(X^F_k - X^0_k)^2 + (Y^2_k - Y^0_k)^2 + (Z^F_k - Z^0_k)^2] + \sum_{1}^{m}{}_{ki}\, C_2\ (S^F_{ki} - S^0_{ki}) \rightarrow \text{minimum}$$

C_1 and C_2 are weighting coefficients and give a proportional weight to the wish to obtain a specific solution, in which the conditions of geometry and stresses prevail.

The restraints are represented by the conditions of equilibrium (4). The number of these conditions is 3 n, n being the number of internal nodes.

Other secondary restraints may be given, such as:—

- Conditions of loads

$$S^F_{ki} = \text{constant} \qquad (5)$$

- Length conditions

$$L^F_{ki} = \text{constant} \qquad (6)$$

where

$$L^F_{ki} = \sqrt{\Delta^F X^2_{ki} + \Delta^F Y^2_{ki} + \Delta^F Z^2_{ki}} \qquad (7)$$

These conditions prove very useful for structures subjected to strict requirements concerning the interior volume and the useful height.

The objective function must minimise the distance between the allowable pneumatic surface of equilibrium (F) and the surface chosen in the preliminary design (O).

In conclusion, one must find $3\,n+m$ unknown quantities: the 3 n co-ordinates of the nodes (k) and the m loads in the bars (ki).

A.5. *The Interactive Calculation and the Verification of the State of Stress and Displacements*

The interactive method of calculation assumes special importance in the design of pneumatic structures. The advantages, which concern mainly the saving of time and the minimal cost, are shown by the diagram of Figure 3.

With this method, it is possible to obtain, for the designs in question, final results "plotted" directly on paper (Fig. 4).

For calculating displacements and stresses in the various cases of external loads, mainly from the action of wind and snow, a method which does not take into account the elasticity of the material was adopted.

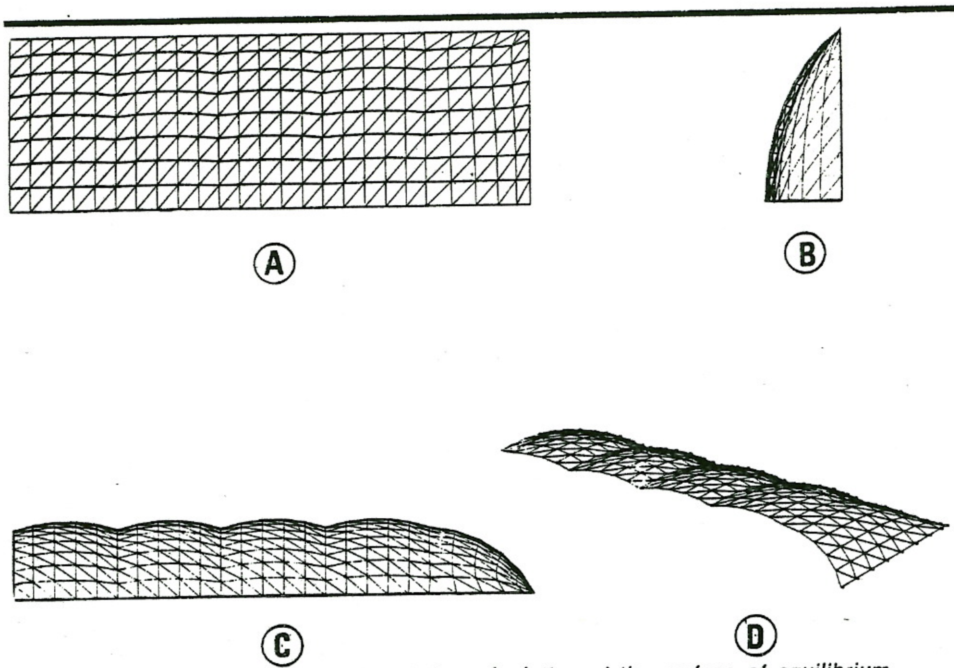

Fig. 4. Direct plotting of the results of the calculation of the surface of equilibrium at state 0 of an inflated pneumatic structure. (A) Plan view.
(B) and (C) Side and front semi-elevations.
(D) Axonometric projection.

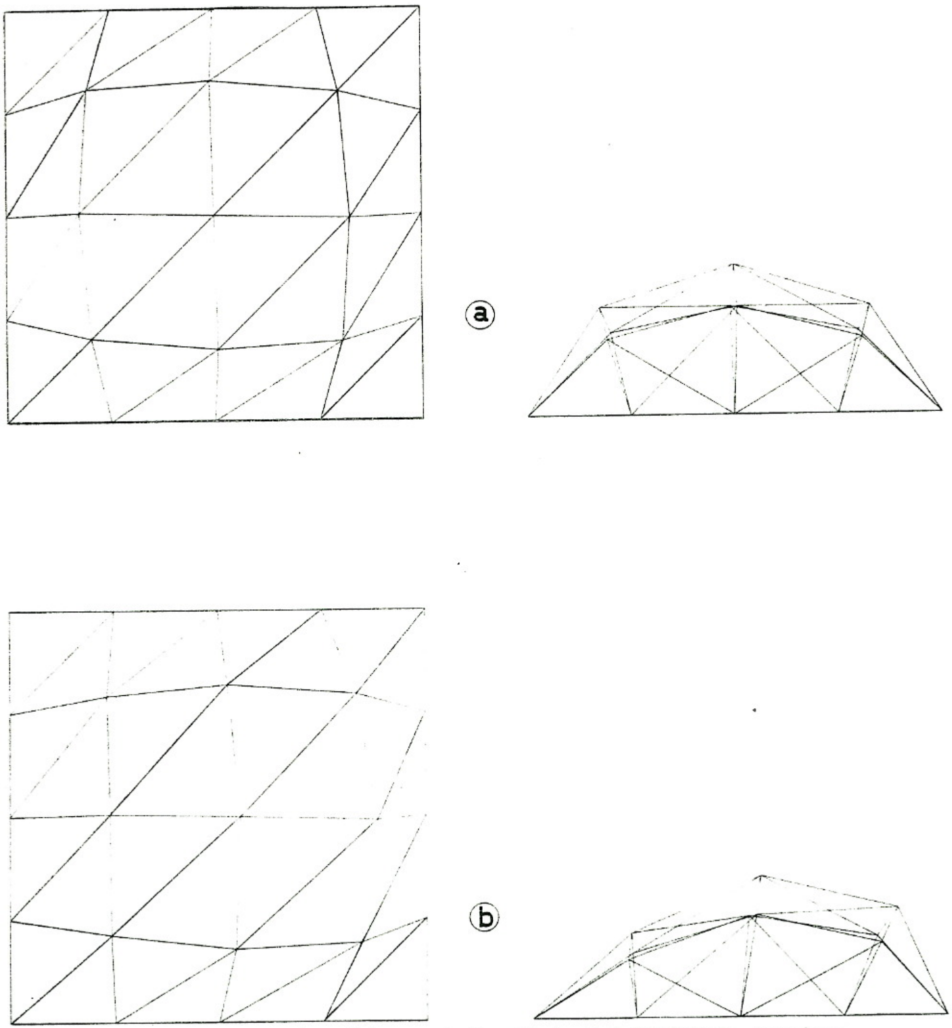

Fig. 5. Graphic representation (plan and elevation) of a pneumatic structure. (A) At the state (0) (dead weight and internal pressure). (B) Subjected in addition to the wind blowing parallel to one of the sides of the base.

In fact, in this problem, the displacements are large, whilst the elongations and expansions of the material are small.

If one studies the action of exterior loads by using the method of nonlinear equations of equilibrium, considering the sizes in question, one meets with problems of numerical instability and results which are sometimes unexpected and unsatisfactory for equilibrium and continuity.

It may be found that the effect of these loads produces only slight variations in expansions, which can be neglected when finding the state of stress and displacements.

As a result, the problem occurs as follows: to keep constant, in the different load cases, the lengths of the imaginary bars which constitute the lattice outlining the pneumatic structure.

By using the same numerical process as before (see A.4) with the conditions (6) relating to the lengths of the bars in the original state (the structure being subjected only to internal pressure), one may determine kinematically the new position of equilibrium of the structure. It is obvious that the bars where the stress $S<0$ are omitted automatically in the interactive method.

An article about this matter is being printed [9].

B. Roofing of an Exhibition Hall in Milan. System with Parallel Cables Anchored to the Ground

The pneumatic structure, of very modern design, covers an area of 50 × 150 m at an average height of 10 m (Fig. 6). High-strength steel cables are arranged crosswise at regular intervals of 15 m, so as to reduce the membrane loads in the fabric which forms the roofing.

The result of binding the cables is an appreciable improvement in the behaviour of the polyester and P.V.C. membrane, reducing stresses both in the layer subjected to internal pressure and in the layer on which the external loads are exerted.

The design of this type of pneumatic membrane is made in two stages. In the first, the balanced geometric configuration under the effect of internal pressure and self weight is determined. In a second stage, the final form of the scheme and checking of the structure under stress from external loads are based mainly on foreign regulations, while waiting for the appropriate Italian standards to appear [7].

B.1. *Materials*

The roof membrane has the following characteristics:—

- Special polyester fabric, covered

on both sides with P.V.C. The latter is combined with non-inflammable materials which render it self-extinguishable and with materials having anti-ultraviolet ray and antistatic properties;
- Weight per m^2: 1,000 g;
- Thickness: 0.85 mm;
- Breaking load, measured on a strip 50 mm in width: 430/480 kg;
- Breaking load, with tearing started: 100 kg.

The membrane is made up of pieces 2 m wide, joined by high-frequency (radio-frequency) welding by means of electrodes 40 mm in width. This guarantees the constancy of the breaking load even at the joins.

All along the perimeter of the membrane, a reinforced hem, made of a similar fabric, but having a breaking load of 600 kg, a galvanised-iron tube 1 1/2″ in diameter is inserted, serving to stiffen the edges.

The cables are of the spiroidal type and 28 mm in diameter, with a breaking load $\geqslant$ 160 kg/mm^2. They have one end fixed and one end adjustable.

B.2. *Pressurisation and Heating Equipment*

It is essential, for the good order and safety of the "pressure-controlled" roof, that the pressurisation appliances should be correctly sized. If one may venture this comparison, these appliances are to be considered as the "heart of a living organism". It is on their efficient working that the "health" of this organism and good working conditions at all points of its surface depend. The static pressure for which the structure was calculated must be provided.

This is why particular care was taken to choose the most suitable and reliable equipment.

It comprises two hot and cold air generators, placed at the two opposite ends of the hall. Provision is made for two changes of air per hour and a static pressure of 28/30 mm of H_2O.

The heating power of the two generators together is 2,000,000 Cal/hour, which makes it possible to overcome a difference in temperature of 25° and to maintain a relative humidity of about 60 %.

Provision was made for the recovery of 50 % of the hot air. The result is a considerable saving of energy and a better control of the static pressure, which can be automatically adjusted according to the atmospheric conditions at the time.

The fans are of the low-speed centrifugal type and are each driven by a three-phase 25 HP motor.

Fig. 6. The exhibition hall at Milan (Italy). Side view of the entrance. Note the parallel cables placed every 15 m.

The diesel-oil burners are the PC 100 model, auto-fed. All the apparatus is fitted with the attachments necessary for automatic control of the atmospheric conditions in the hall.

There is also an automatic safety device in case of emergency. In fact, exceptional atmospheric conditions or interruption of the externally supplied current could cause considerable damage to the whole system, if the interior static pressure were not guaranteed by special fans, ready to intervene automatically at the first sign of danger.

It is vital that this intervention should be immediate, for in the space of time, even if it were only a few seconds, which could elapse between interruption of the main supply and coming into force of the emergency supply, the hall could find itself at the mercy of the action of the wind, with risks of tearing.

In order to obviate this danger, the emergency system is twofold, one supplied by direct current, the other by a diesel engine.

It comprises the following:—

(*a*) A direct-current (110 V) automatic intervention unit consisting of a double-exhaust centrifugal fan, with high efficiency and low speed (600 rpm) driven by a 15 HP motor, completed by an automatic operating rheostat, by a rectifier and by a stationary battery, which ensures independent working for 2 hours;

(*b*) A safety device comprising a fan with the same characteristics, driven by a 20 HP diesel engine, with automatic gear selection in the event of interruption of the external current.

The main purpose of the battery unit is to intervene immediately while the other unit reaches ideal operating conditions, after which it functions at a specific pressure, whilst the diesel-engine unit continues to operate until the return of the external current.

Furthermore, an anemometer situated outside automatically initiates action of the battery unit, each time the wind exceeds a predetermined speed.

With this duplicate standby equipment, the possibility of mishaps due to external factors can be considered as practically nil.

C. Pneumatic Structures with Flattened-section Reinforcement Cables

Since completion of the United States pavilion at the Osaka 1970 exhibition (Japan), the research carried out in this field in the United States has culminated in the building of six pneumatic structures with flattened-section cables, the characteristics of which will be found in Table 1. On plan, these structures have varied shapes,

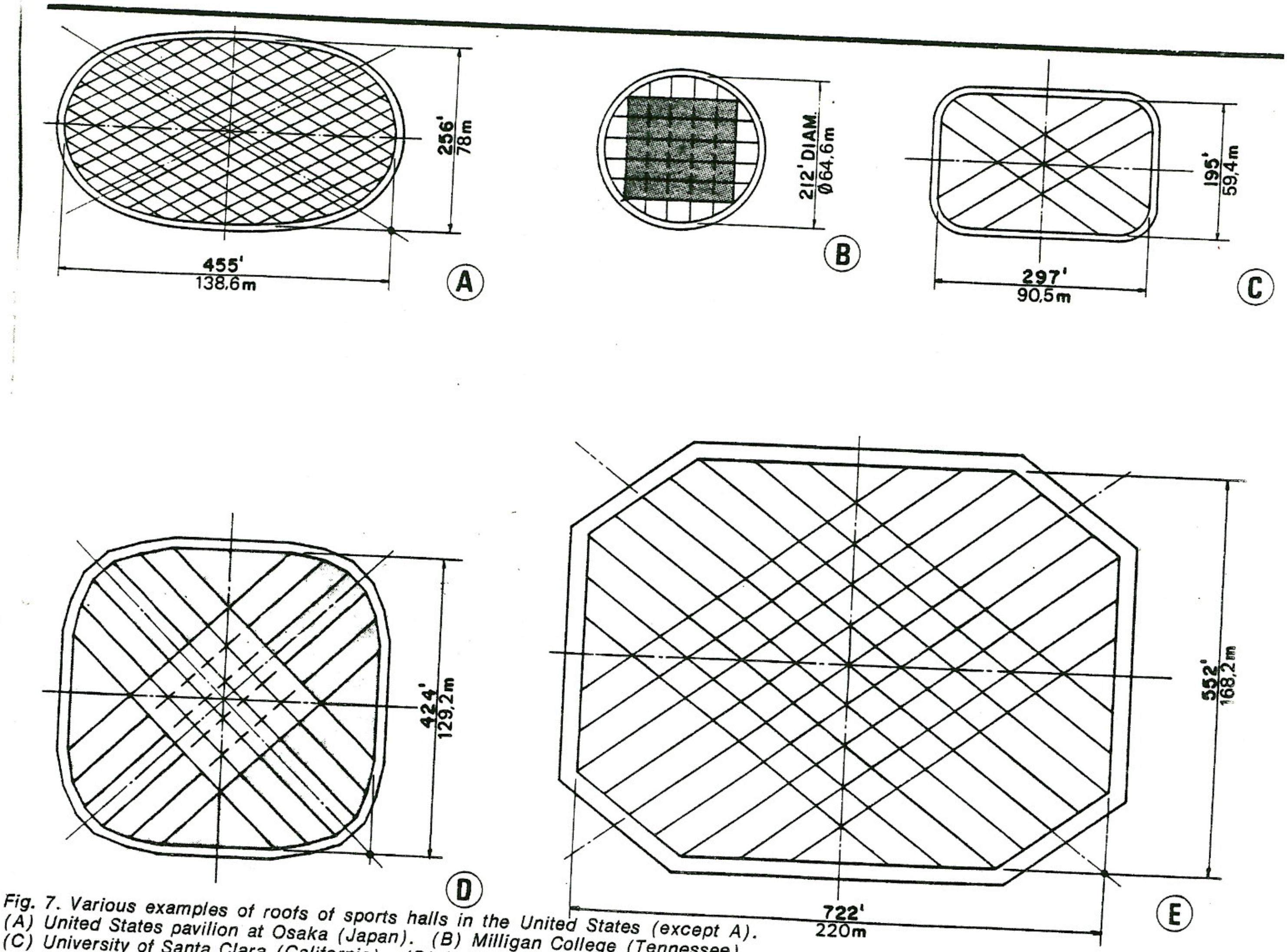

Fig. 7. Various examples of roofs of sports halls in the United States (except A).
(A) United States pavilion at Osaka (Japan). (B) Milligan College (Tennessee).
(C) University of Santa Clara (California). (D) University of the North of Iowa.
(E) Stadium of Pontiac (Michigan).

from a circle 64 m in diameter to a rectangle 168 × 220 m with cut-off corners (Fig. 7).

Each of these roofs is provided with a horizontal compression ring, whilst the deflection at the crown is of the order of 1/10 to 1/20 of the span.

Provided that the internal pressure is maintained at levels of between 20 and 30 kg/m^2 and that the weight of the roofing does not exceed 3 to 5 kg/m^2, the structure remains balanced.

There is not therefore any problem of instability, and it is thus possible to reduce deflections to the minimum compatible with the requirement of rainwater drainage. As the deflection is reduced, so the action of the wind is lessened.

Since the ring is situated in a horizontal plane, there is symmetry in relation to this plane between the configuration of the deflated structure and that of the inflated structure. It is possible that, deflated, it can hang freely, without damage either to property or to people, so as to reduce its dependence upon the mechanical system.

As the deflection is reduced, the horizontal loads which are to be absorbed by the compression ring are considerable and it is necessary to resist these loads with as much strength as possible.

This will be the case if the ring has a "funicular" behaviour, especially if the moments under the effect of the internal pressure only are nil. The traditional circular structure fully meets this requirement when it is subjected to radial stresses of constant intensity. But if these stresses are transmitted by radial cables, a ring of tension must exist in the centre. The weight of this ring will be such that, in a pneumatic structure, it will create an area of depression, possibly inducing sagging of the roofing, which will cause an accumulation of water. This solution is not therefore practical.

As an alternative to the radial cables, use is made of a system of orthogonal cables with constant horizontal loads (Fig. 7). This arrangement of cables corresponds to the traditional solution, using membrane, for a field of uniform forces in the orthogonal directions.

Likewise, if the fields of orthogonal force are uniform, but unequal ($6x \neq 6y$) it may be established that an elliptical ring having the large axis a and the small axis b parallel to the directions of the main stresses, is "funicular" for this field of forces, if

$$\frac{6x}{6y} = \frac{a^2}{b^2} \qquad (8)$$

In this case, the problem is to determine the geometric form of the ring, taking as data the loads transmitted by the roofing membrane.

If the ring is symmetrical in relation to the axes x and y and if the cables or the field of forces are parallel to these axes, the designer can establish, to a close arbitrary constant, the distribution of the horizontal components of the loads in the cables, in consideration of the use of the equilibrium equations in each of the quadrants.

Provided that the external loads are vertical or symmetrically distributed, the horizontal reactions at the ends of the cables are equal and opposite, and the funicular solution for a quadrant applies to all the others.

Fig. 8. Sports hall at Santa Clara (United States). Exterior view.

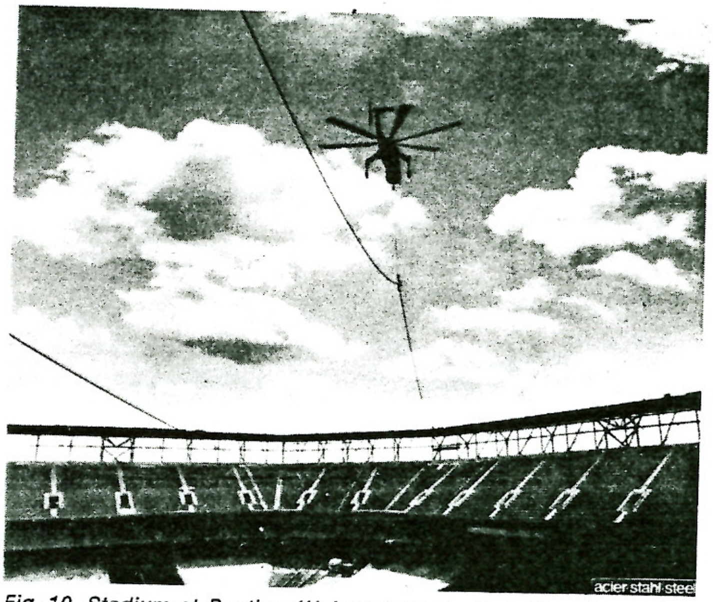

Fig. 10. Stadium at Pontiac (United States). Installing the cables.

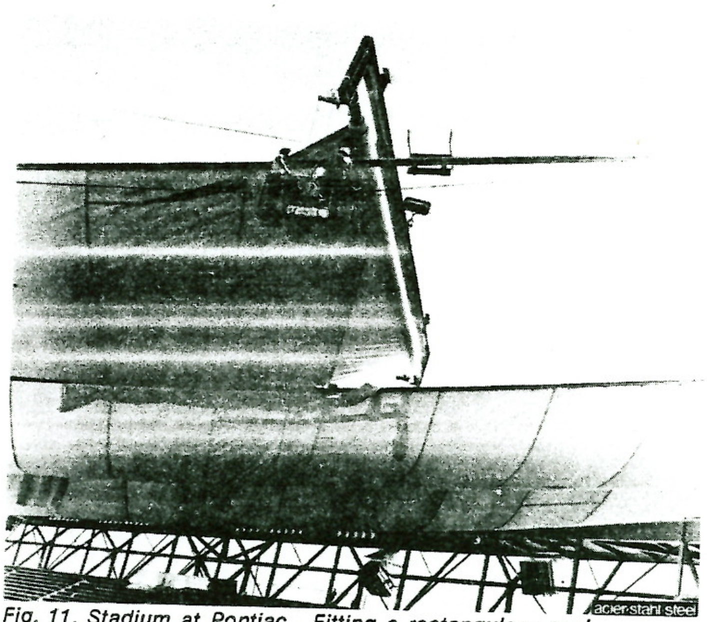

Fig. 11. Stadium at Pontiac. Fitting a rectangular panel and fixing to the cables.

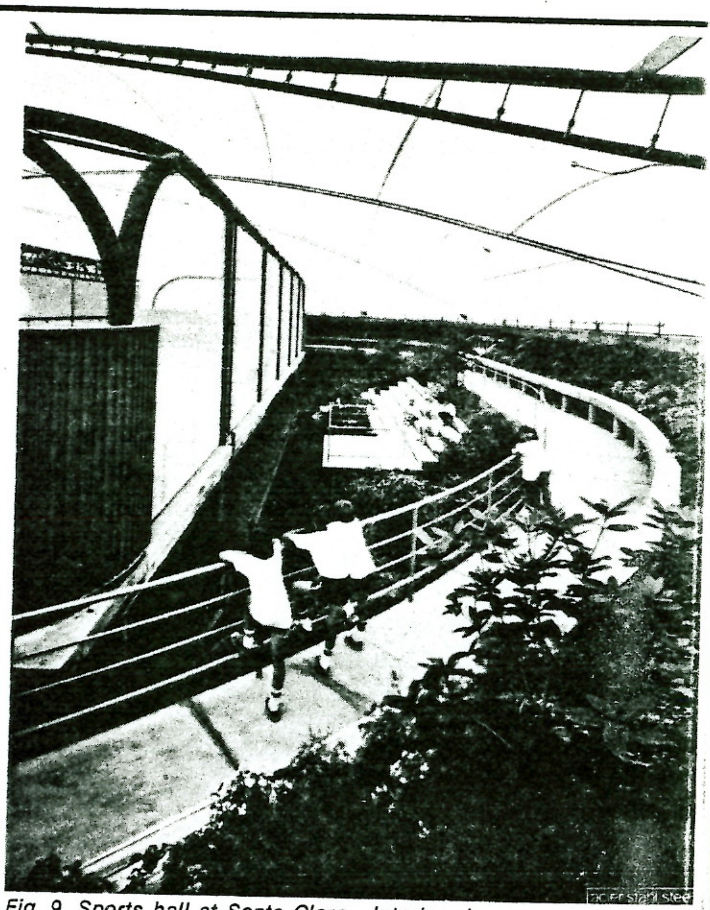

Fig. 9. Sports hall at Santa Clara. Interior view.

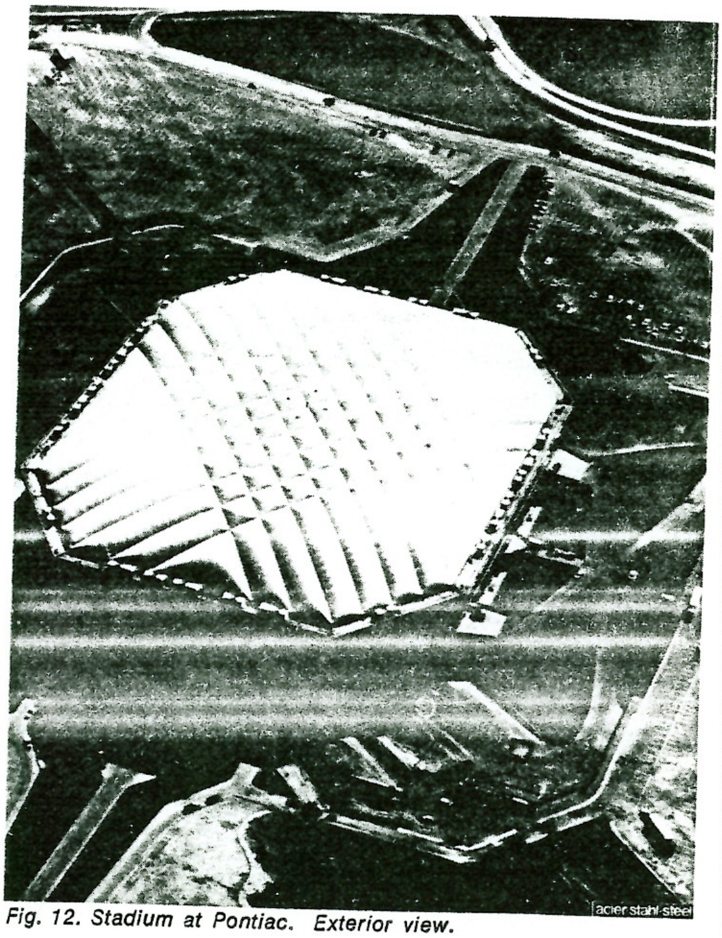

Fig. 12. Stadium at Pontiac. Exterior view.

Table 1. *Main Characteristics of the Elliptical Inflatable Structures* (see Fig. 7).

Installation	U.S.A. Pavilion Osaka, Japan	Milligan College, Johnston City Tennessee, U.S.A.	Santa Clara University, Santa Clara, California, U.S.A.	University of North Iowa, Cedar Falls, Iowa, U.S.A.	Pontiac Stadium, Pontiac, Michigan, U.S.A.
Dimensions (m)	138.6 × 78.0	Ø 64.6	90.5 × 59.4	129.2 × 129.2	220.0 × 168.3
Height in roof (m)	7.0	4.8	7.2	14.6	15.2
Spacing of cables (m)	6.0	9.0	12.0	12.9	12.6
Diameter of cables (mm)	38.1 to 57.3	38.1 and 2 of 27.0	47.6	73.0	79.4
Nature of fabric	PVC-glass 1 layer	Teflon-glass double, insulated; also 1 layer	Teflon-glass 1 layer	Teflon-glass partly double	Teflon-glass single layer
Wind suction (lb/ft²) (N/m²)	30.0 1,464.6	15.0 732.3	12.0 585.9	15.0 732.3	15.0 732.3
Snow load, roof deflated (lb/ft²) (N/m²)	12.0 589.9	20.0 976.5	12.0 585.9	30.0 1,464.6	12.0 585.9
Weight of roof (lb/ft²) (N/m²)	1,25 61.0	1.0 48.8	0.9 43.9	1.0 48.8	1.0 48.8
Air supply output (ft³/min) (m³/min)	120,000 3,398	130,000 3,681	400,000 11,326	370,000 10,477	3,500,000 99,109
Maximum accommodation of people	5,000	1,800	5,000	18,000	80,000
Translucency of roof	6 %	0 % and 14 %	14 %	average 11 %	average 8 %

Once these horizontal components are known, if the cables are contained in parallel planes, the vertical ordinates of the system may be determined by solving a series of simultaneous equations, based on the static balance of each of the nodes.

If this method is applied to a circular or elliptical ring, with uniform loads, it is established that the form of the cables will be parabolic.

If, on the contrary, this method is used for a "super-ellipse", where

$$\left[\frac{x}{a}\right]^M + \left[\frac{y}{b}\right]^M = 1 \qquad (9)$$

with $M > 2$ (Fig. 13),

it may be seen that the dimensions of the cables increase in a significant manner and that the cables nearest to the axes of the ellipse come to have a point of inflexion. The roofing assumes a shape which is not the best one for draining away water.

We must then ask ourselves whether there exists another direction in which the cables could be arranged, so that the ring still appears as "funicular", but without the curves of the cables changing sign.

A careful mathematical study shows that this alternative exists and, more precisely, when the cables extend in directions parallel to the diagonals of the rectangle of which we have spoken before. After finally determining the relationship between the arrangement of the cables and the geometry of the ring, in the state of stress where the latter is "funicular" one must determine the effects of the loads whatever their distribution.

When the network of cables does not have triangular meshes and we are in the field of considerable deformations under the effect of accidental loads, the problem to be solved becomes of the non-linear type.

The results obtained, in solving the problem by application of the equilibrium equations, must be interpreted with a great deal of care.

The fact of obtaining convergence through the control of equilibrium does not guarantee a correct solution. It must also satisfy compatibility, and for this be very cautious in view of the possibility of numerical instability of finite element methods.

After having completed the non-linear calculation of the network of cables, one may go on to that of the loads acting upon the ring, which is no longer "funicular", and to checking this ring for bending moments.

The greatest advantage of this method of construction lies in its moderate cost, and also in the speed and ease of erection.

When the designers have familiarised themselves with this technique and have grown aware of the beauty of interior areas lit by translucid roofing materials, such as Teflon (Fig. 9), and when, furthermore, the methods of recovering energy have improved, further applications may become of interest, and not only for wide spans.

D. Scheme for the Covered Stadium of Pescara

We are going to describe briefly the design produced by the architect Rodrigo Fradeani, who called upon the authors of this article to be consulting engineers for the structure of this stadium.

The basic data were as follows: capacity of 5,000 spectators (which may be increased by occupation of the pit during boxing matches) and possibility of use full-time for the greatest number of participants in the various forms of sport.

At the start, difficult technical and environmental problems were encountered.

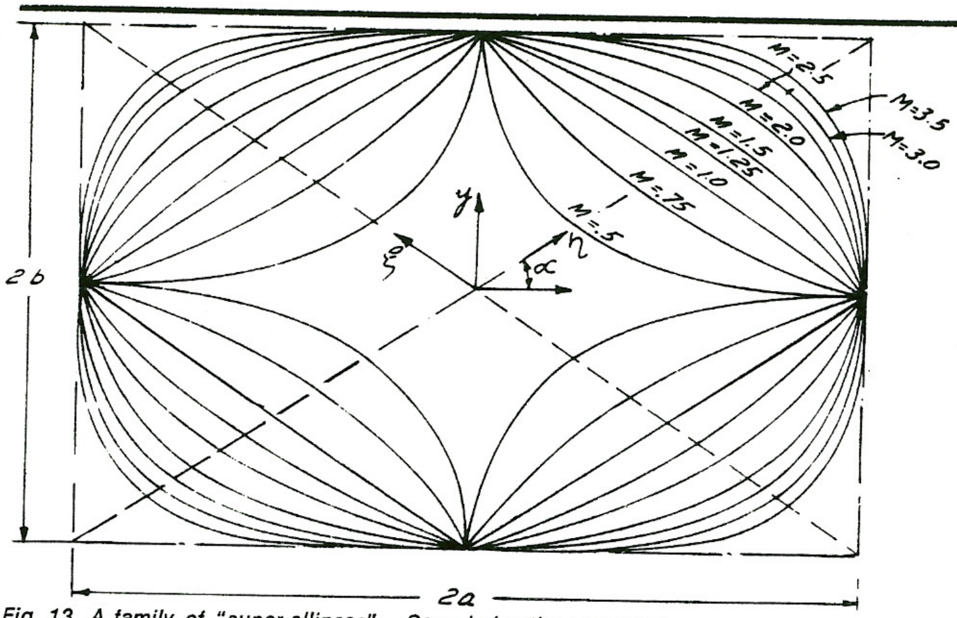

Fig. 13. A family of "super-ellipses". Search for the geometry of the edge ring.

Among the technical problems was, in particular, the nature of the foundations in a soil of poor consistency. Now these foundations had to support a large-span roof which not only had to be economical, but also stand up with the minimum of maintenance to a highly corrosive marine atmosphere.

With regard to the environment, the problem was to insert a construction of such large dimensions in the existing natural framework of a pine forest.

Thus it happened that a pneumatic roof on high-strength steel cables was chosen. The cables are anchored in a reinforced-concrete ring, placed at the top of an embankment which rises to 8 m above the level of the access road. Thus a solution was found, outside, for the problem of insertion in the environment (for the embankment is covered with vegetation), and, inside, that of the foundations, for the tiers rest directly on the slope of the embankment.

The concrete ring is "superelliptical" in shape, with axes of 118.50 m and 74 m. The surface area of the membrane is about 7,555 m^2. The cables are arranged parallel to each other every 6 m, and have a diameter of 50 mm.

The pressurisation installation will be made up of centrifugal turbines, with spare turbines in case of breakdown. There will also be two generating sets to provide for interruptions in current.

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